Six Sigma

Statistics for Green Belt Certification

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Content

Analysis Strategies

In the DEFINE-Phase the critical output variables (Y= CtQ´s= Problems) will be identified …

SIPOC/ Process-Mapping-Analysis

S	Input	Process		Output	C
		Method	Ressource		
	Influence	Influence	Influence	Problem	
	XI1	XM1	XR1	Yt1_Q_1_n	
	XI2	XM2	XR2	Yt1_T_1_n	
	XIn	XM3	XRn	Yt1_R_1_n	

C&E-Matrix

		Input	Method	Resource
		XI1_n	XM1_n	XR1_n
Output	Yt1_Q_1_n	❶	❷	❸
	Yt1_T_1_n		❹	
	Yt1_R_1_n	Hypothesis		❺

Hypothesis

Relationship:	If X, then Y
	The… X, the… Y
Difference:	in: Y between: Xn /Xm/ …

Data Collection

Variable	Scale	Scale Level	Hypothesis	Chart	Perf.-Ind.	Test
Yt1_Q_1_n	Error Type	nominal	❶❷❸	Pareto-Diagram		
Yt1_T_1_n	Cycle Time	cardinal	❹	Histogram	Sigma Level	Correlation
Yt1_R_1_n	Rating	ordinal	❺			Mood-Median
XI1_n		cardinal	❶			
XM1_n		ordinal	❹			
XR1_n		nominal	❺			

… and in the ANALYSIS-Phase the relevant influences (x) will be identified

Procedure to prepare and perform Statistical Analyses and …

Measure

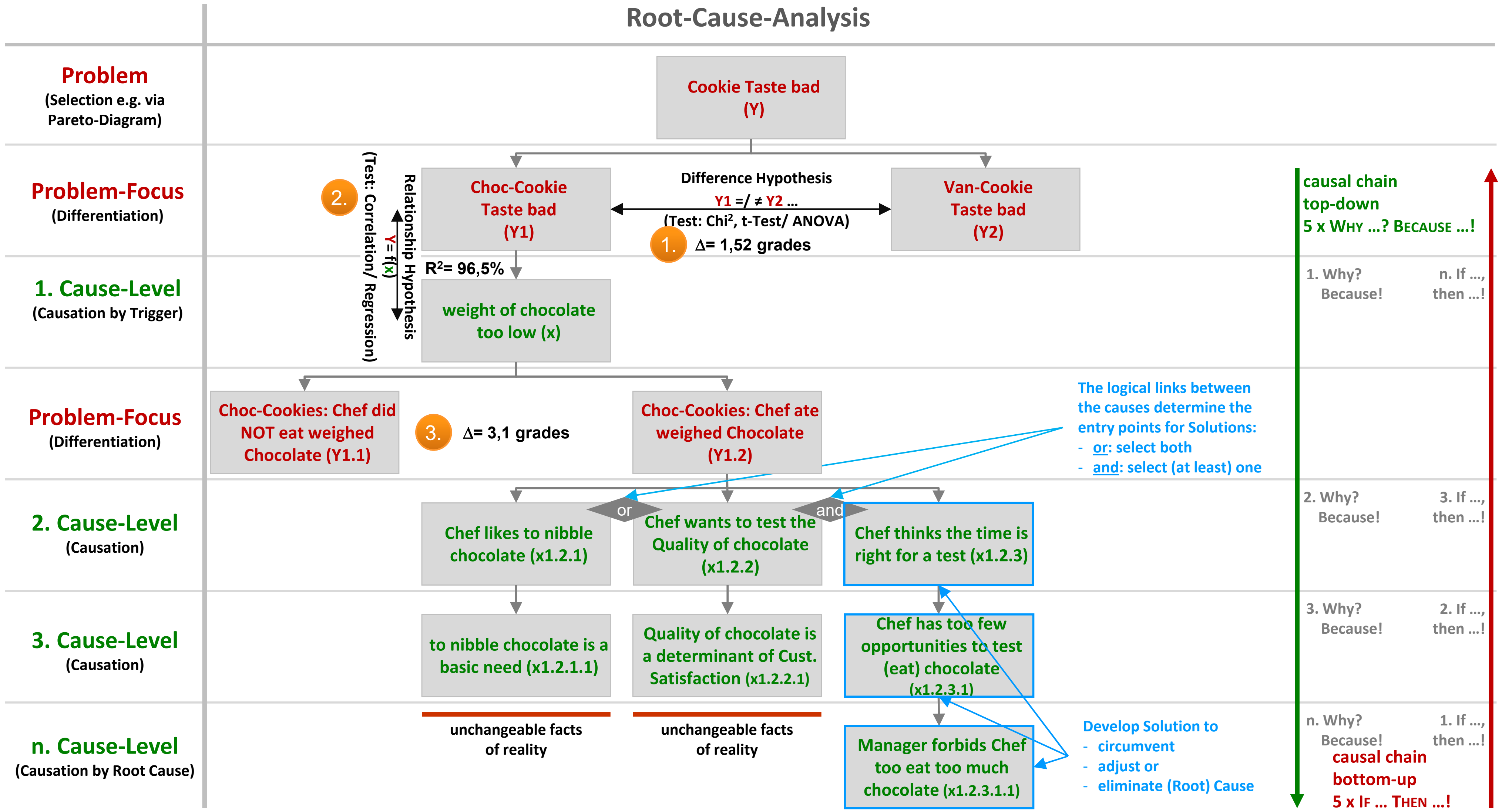
1. Operationalize Measurands (x/Y)
2. Determine Scale Level of Measurand (nominal, ordinal, cardinal)
3. Formulate Hypothesis
4. Select Statistical Test
5. Determine significance level (α= 1%/ 5% / 10%)
6. Calculate Power/ Sample Size
7. Check Measurement System
8. Organize Data Collection
9. **Collect Data**

Analyse

10. Check plausibility of Data (Distribution, Min/ Max)
11. Perform Statistical Test
12. Check the results:
    - Statistical Significance (p > α: H_0 **or** p<α: H_A)
    - Practical Relevance/ Value (size of Difference ($Y_1 - Y_2$ **or** Strength of Relationship $Y = f(x)$)
13. Interpret results
14. Analyse Root-Causes
15. Identify anchor points for Solutions in Improve

… and its consequences for IMPROVE

Root-Cause-Analysis: combines focusing by differentiation and analysing of causal relations



... as well as quantitative statistical and qualitative logical reasoning to identify the Root-Causes

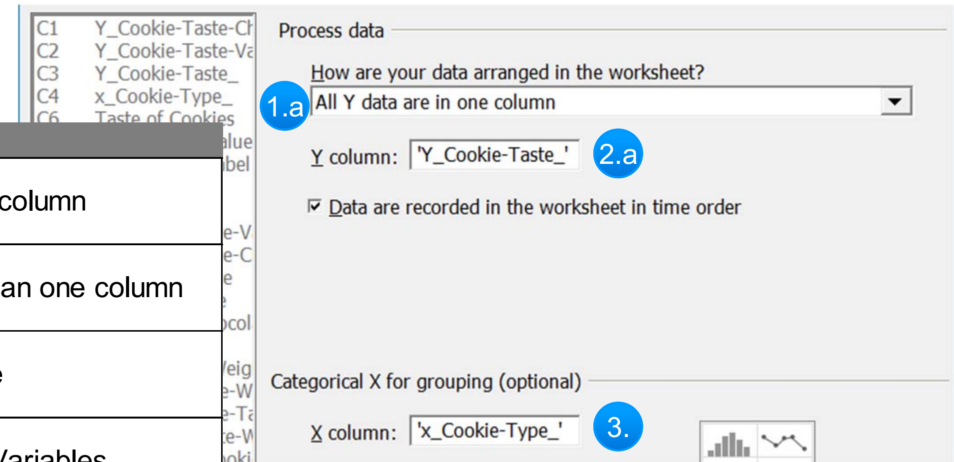
Hints and typical errors in Minitab

Grouped data can be arranged in two alternative ways:

2.a C3	3. C4-T
Y_Cookie-Taste_	x_Cookie-Type_
1	Y_Cookie-Taste-Chocolate_1
2	Y_Cookie-Taste-Chocolate_1
3	Y_Cookie-Taste-Chocolate_1
4	Y_Cookie-Taste-Chocolate_1
5	Y_Cookie-Taste-Chocolate_1
6	Y_Cookie-Taste-Vanilla_1
7	Y_Cookie-Taste-Vanilla_1
8	Y_Cookie-Taste-Vanilla_1
9	Y_Cookie-Taste-Vanilla_1
10	Y_Cookie-Taste-Vanilla_1

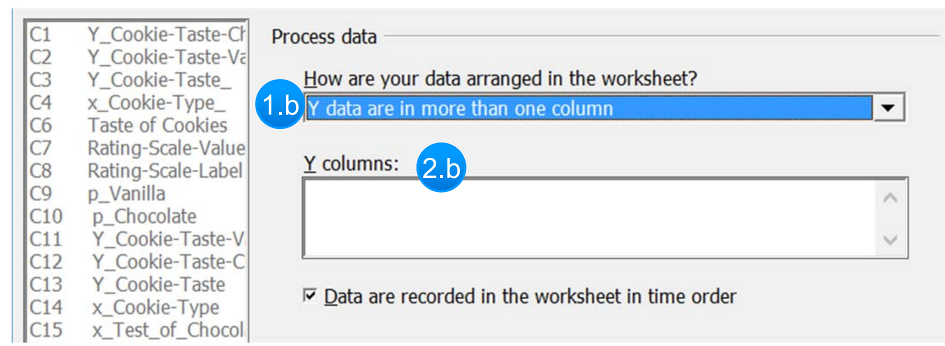
1.a

Dialog	
1.a	All Y data are in one column
1.b	Y data are in more than one column
2.a	select one Y-Variable
2.b	select all relevant Y-Variables
3.a	select grouping Variable x for Y



2.b C1	2.b C2
Y_Cookie-Taste-Chocolate_1	Y_Cookie-Taste-Vanilla_1
1	6
2	7
3	8
4	9
5	10

1.b



1.a All Y data in one column with a grouping x - or - 1.b Y data grouped in different columns

More hints and errors

- The type of data must be appropriate for the procedure. Minitab offers different types of data, e.g.: a) numeric, b) text and c) date/ time. The type of date in a column is indicated in the top row.
  The data type can be changed, if necessary, with: Data/ Change Data Type …

- Missing Values are indicated by:
  - „*“ for a) numeric and c) date/ time
  - „*Missing*“ for b) text values

- All columns of data, used for the same procedure, must have the same length. If they do not have the same length, then enter a value in the „shorter“ column, one position under the needed length. The empty cells will then be filled with the Missing Value indicator. At least delete this value again. The Missing Value indicator will remain and the columns have the same length.

- If you enter data manually, then you can change the direction for entering data – in a column or in a row – by a click on the arrow.

↓	C1	C2-T	C3-D
	a_Number	b_Text	c_Date
1	1	A	01.01.2016
2	2	B	02.01.2016
3	3	C	03.01.2016
4	4	D	04.01.2016
5			

↓	C1	C2-T	C3-D
	a_Number	b_Text	c_Date
1	1	A	01.01.2016
2	*	*Missing*	*
3	3	C	03.01.2016
4	4	D	04.01.2016
5			

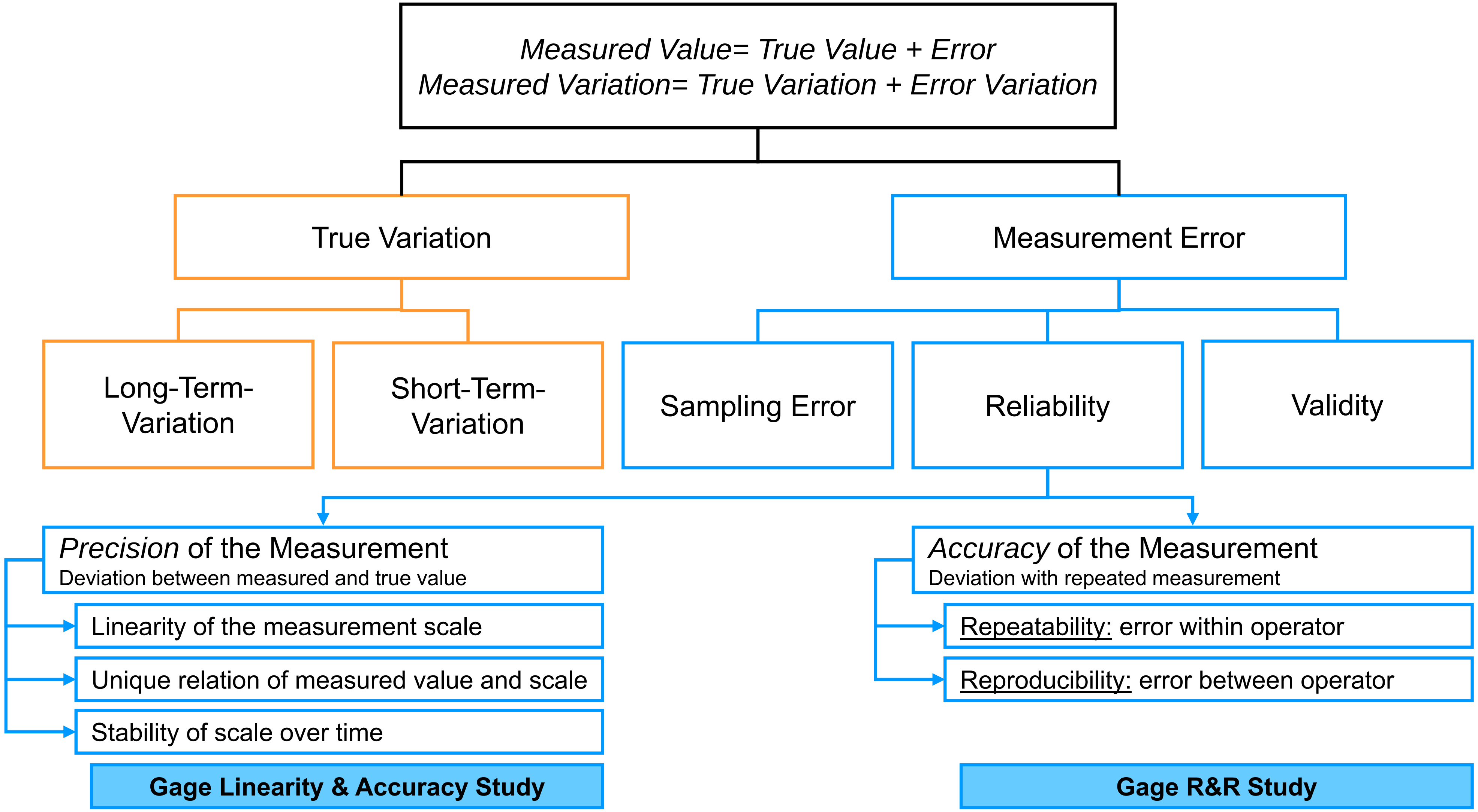
↓	C1	C2-T	C3-D
	a_Number	b_Text	c_Date
1	1	A	01.01.2016
2	*	*Missing*	*
3	3	C	03.01.2016
4		D	04.01.2016
5			

→	C1	C2-T	C3-D
	a_Number	b_Text	c_Date
1	1	A	01.01.2016
2	*	*Missing*	*
3	3	C	03.01.2016
4	*	D	04.01.2016
5	1		

…

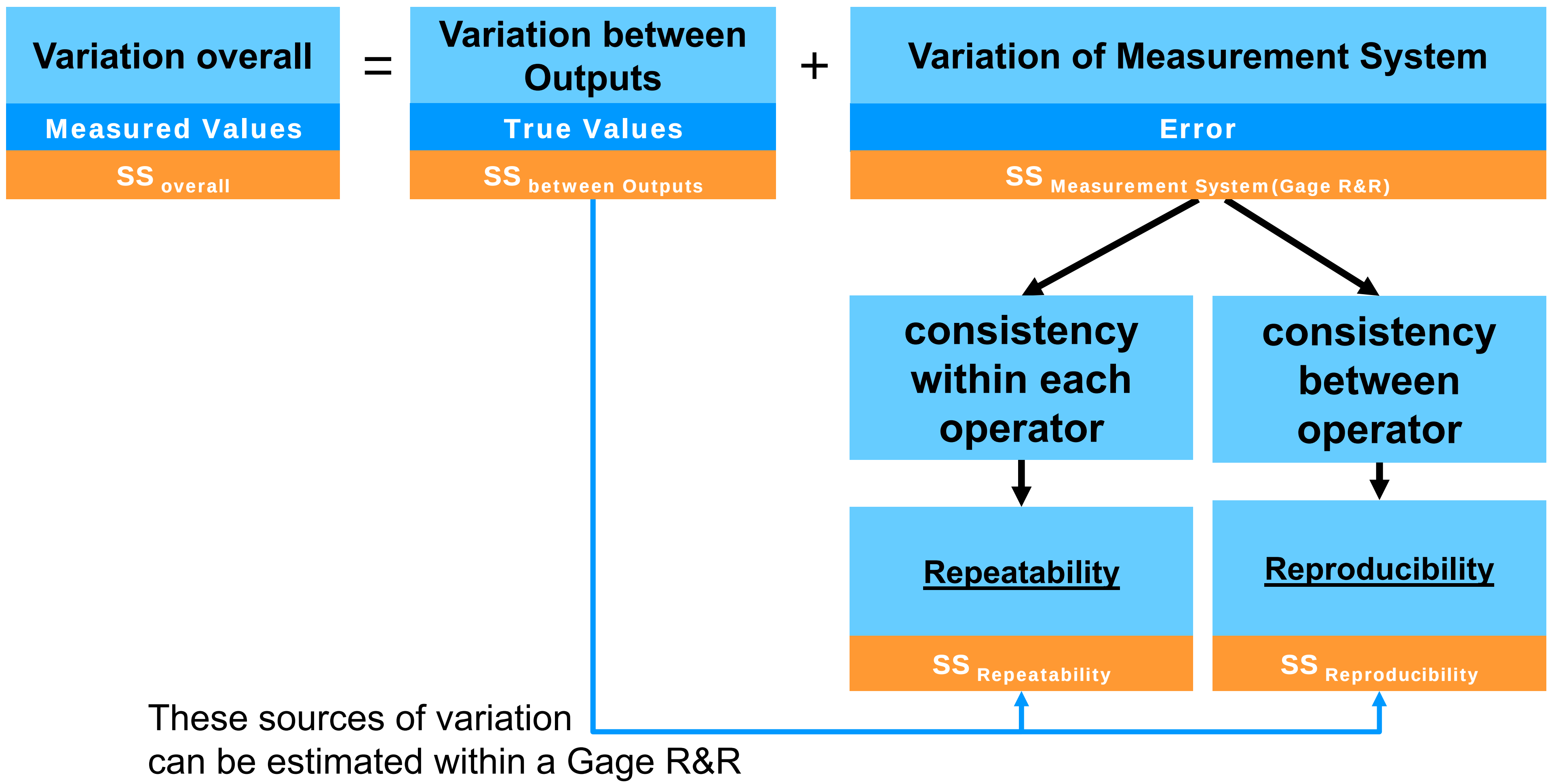
Measurement System Analysis (MSA)

Every measured value is error-prone



The Measurement System Analysis (MSA) helps to reduce the size of error

Separate intentional from unintentional sources of variation

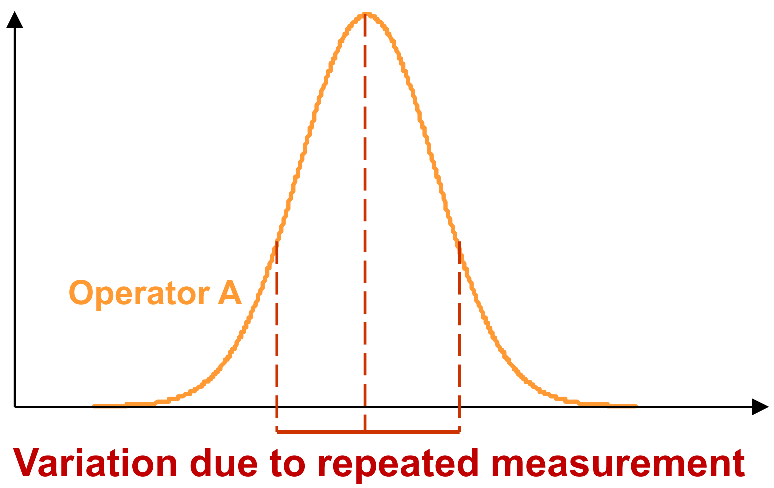


The variation between Outputs should be larger than the variation of the Measurement System

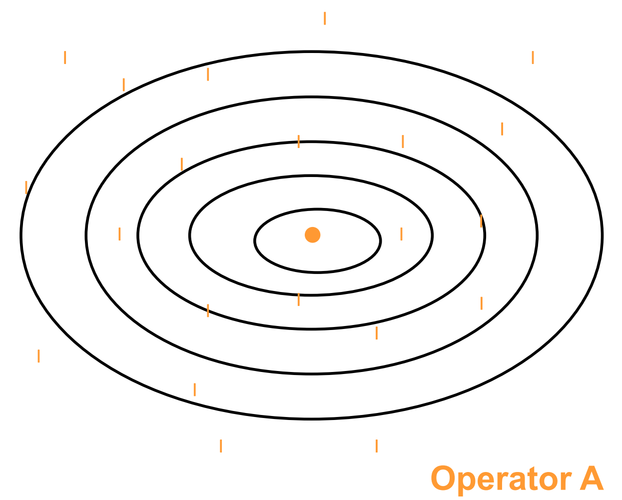
Repeatability

Repeatability:= Variation due to

- repeated appraisal
- same output
- **same operator**
- same measurement system



N repeated appraisals of the same Output (Trials)

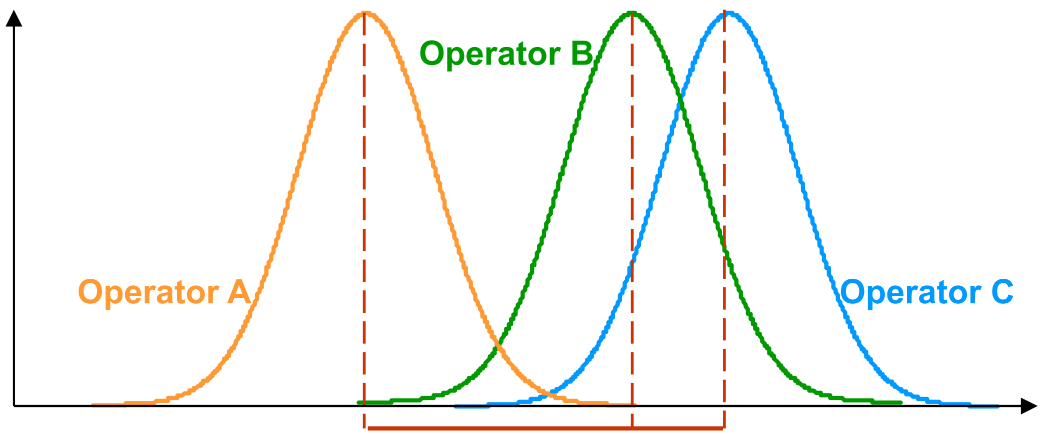


Variation of values due to repeated measurements

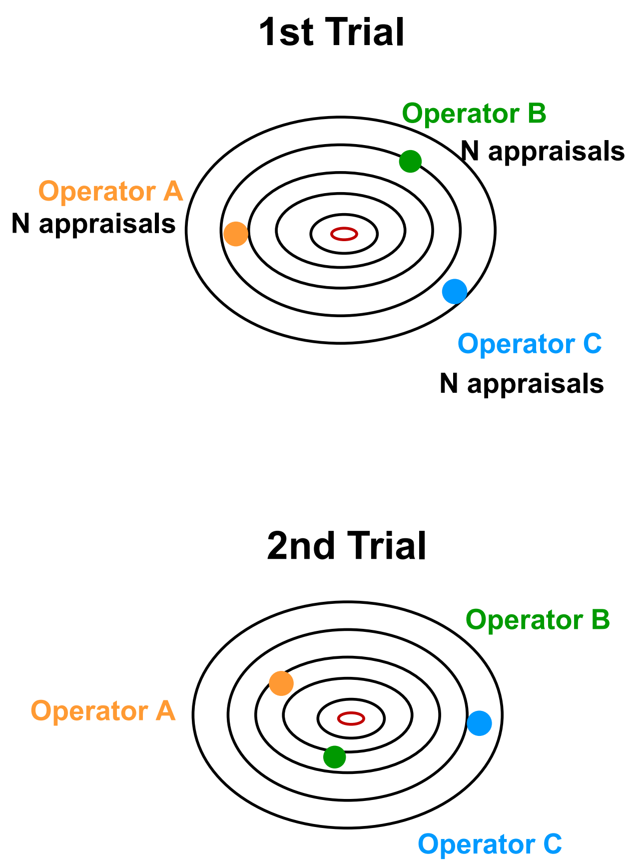
Reproducibility

Reproducibility:= Variation due to

- repeated appraisal
- same output
- **different operators**
- same measurement system

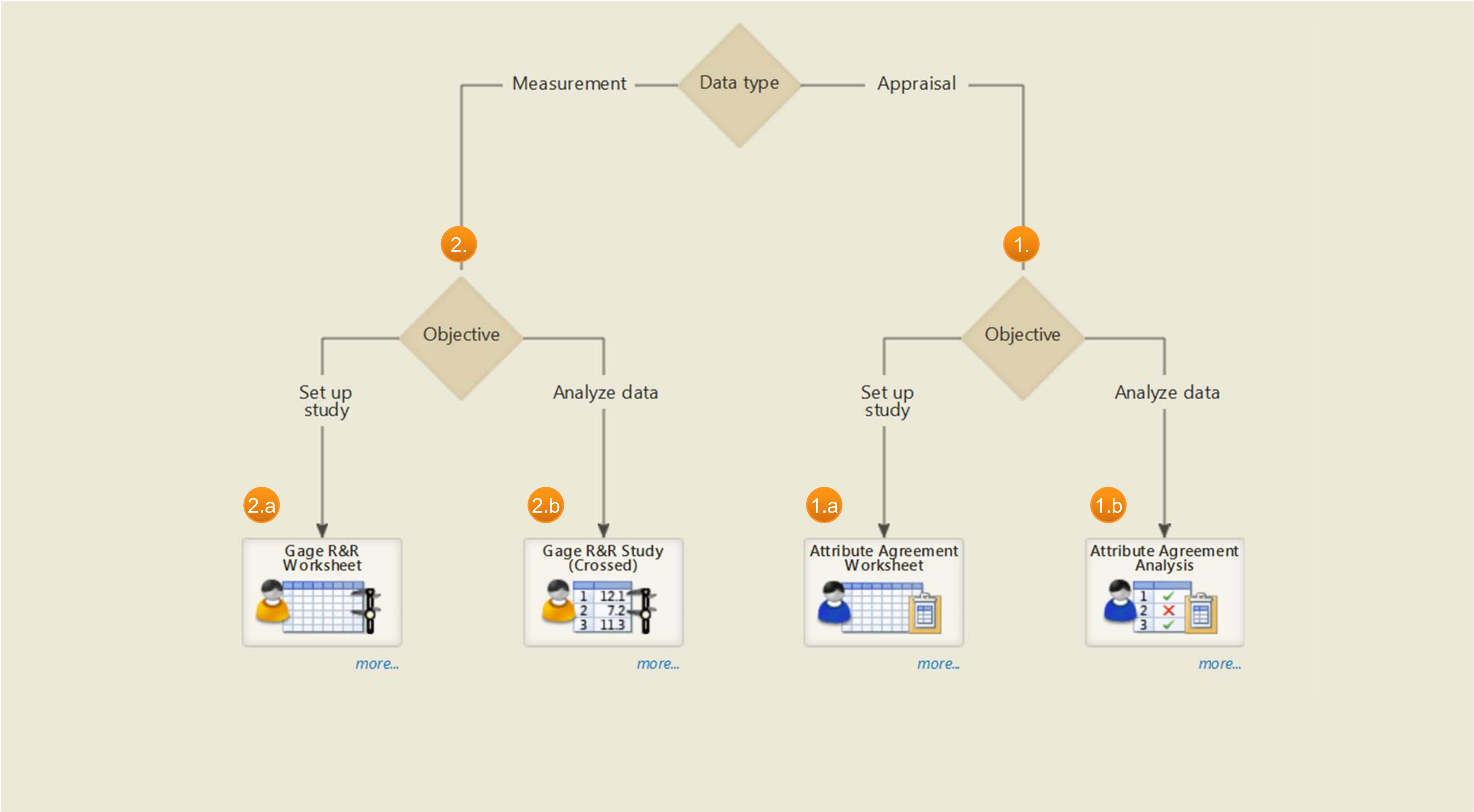


Variation due to reproduced measurement



Variation of values due to reproduced measurements

Minitab Assistant: Measurement Systems Analysis



Attribute Agreement Analysis and Gage R&R Study

1.

Attributive Agreement Analysis: Evaluation of Nominal-Scaled Measurements

MSA

Attributive Agreement Analysis (= attribute gage R&R study)

Purpose

Evaluates the agreement of subjective Nominal-/ Ordinal-Scale based ratings by multiple Operators to determine how likely a Measurement System is to misclassify an Output

Focus

Reliability Repeatability and Reproduceibility (R&R)

Example

Evaluate R&R of Inputs and Outputs of your Process

Evaluate R&R of: supplied (spare) parts, guarantee claims, credit decisions, answers from a call center, produced rods, mounted clips

Evaluate R&R of: M&M´s

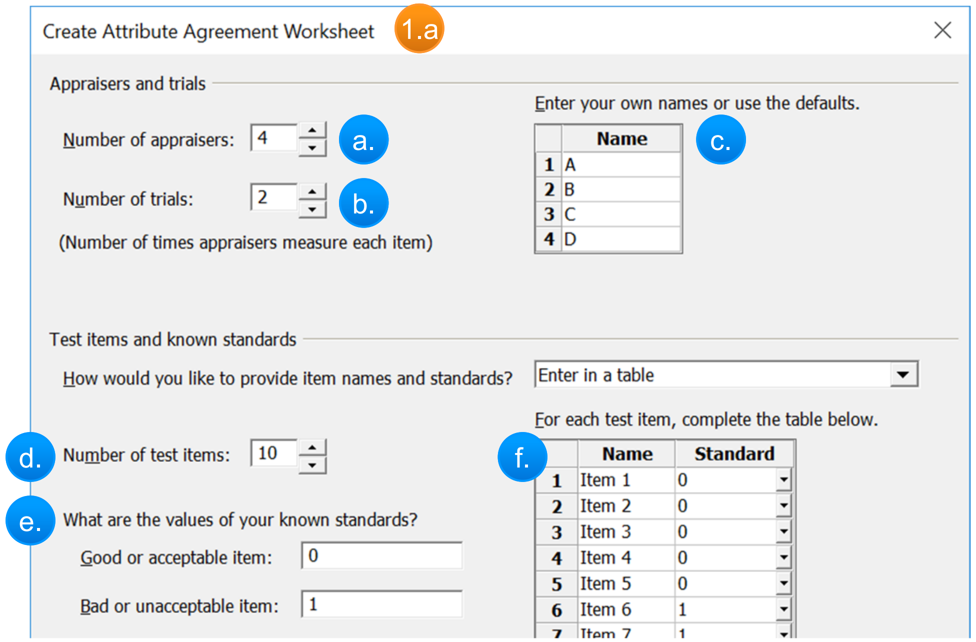
Y	Scale Level
1	nominal/ ordinal
x	Scale Level
./.	./.

Alternative in Minitab Stat Menu

Stat/ Quality Tools/ Attribute Agreement Analysis

note …

Assure a balanced mix of: 0/good and 1/bad items. A small percentage of items of one type reduces the ability to assess R&R



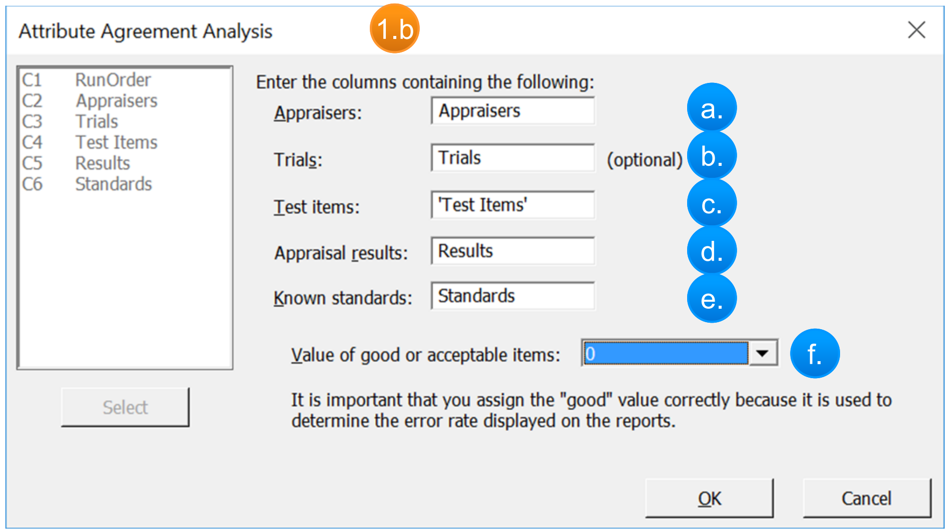
Dialog: Create Worksheet	
a. Number of appraisers	Determine the Operators, which should be included as appraisers into the study; if possible select all involved Operators
b. Number of trials	Select at least 2 trials for each appraiser, good style is 3
c. Name of appraiser	To use the real names of the appraiser supports specific training, that may become necessary; but aka-names might be better, starting a series of MSA´s
d. Number of items	select at least 10 items (5 good/ 5 bad)
e. values for good/ bad	to differentiate the good vs. bad items, attributive and numeric labels can be assigned (My preference: 0= good; 1= bad)
f. classify items	classify the test items according to their attributes; the items will be randomly arranged in the worksheet, to avoid sequence effects;

Example: Evaluate the R&R of good vs. bad Inputs or Outputs (like Decisions or M&M´s)

1.

Attributive Agreement Analysis: Evaluation of Nominal-Scaled Measurements

↓	a. C1	b. C2-T	c. C3	d. C4-T	e. C5-T	f. C6-T
	RunOrder	Appraisers	Trials	Test Items	Results	Standards
1	1	A	1	Item 10	1	1
2	2	A	1	Item 1	0	0
3	3	A	1	Item 7	1	1
4	4	A	1	Item 5	0	0
5	5	A	1	Item 8	1	1
6	6	A	1	Item 2	0	0
7	7	A	1	Item 3	0	0
8	8	A	1	Item 4	0	0
9	9	A	1	Item 6	1	1
10	10	A	1	Item 9	1	1
11	11	B	1	Item 1	1	0
12	12	B	1	Item 5	1	0



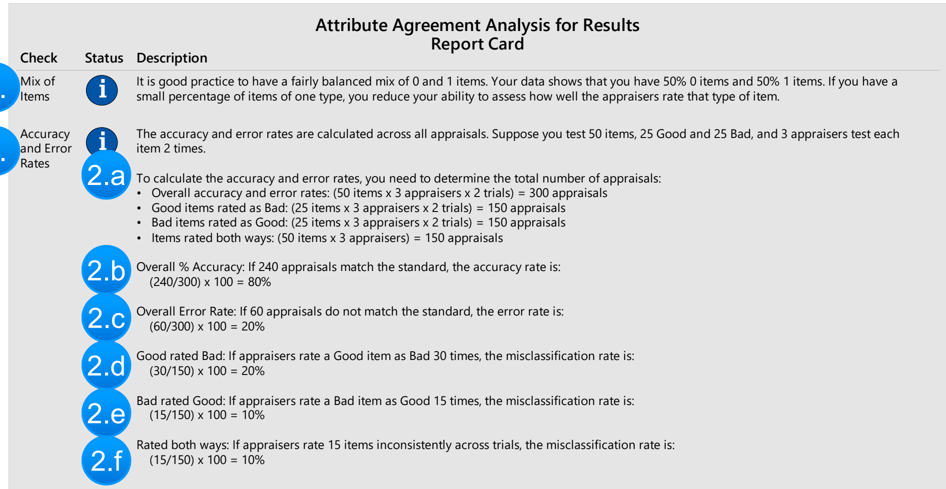
Result: Created Worksheet	
a.	Run order: randomly calculated sequence for evaluating the items; use this order to avoid sequence effects
b.	Name of appraiser for the specific evaluation
c.	specific trials for specific appraiser
d.	item, the appraiser should evaluate in this evaluation
e.	result of evaluation of appraiser; enter the coded answers here, e.g. 0 for good of 1 for bad
f.	previously - by experts - defined standard evaluations of the test items

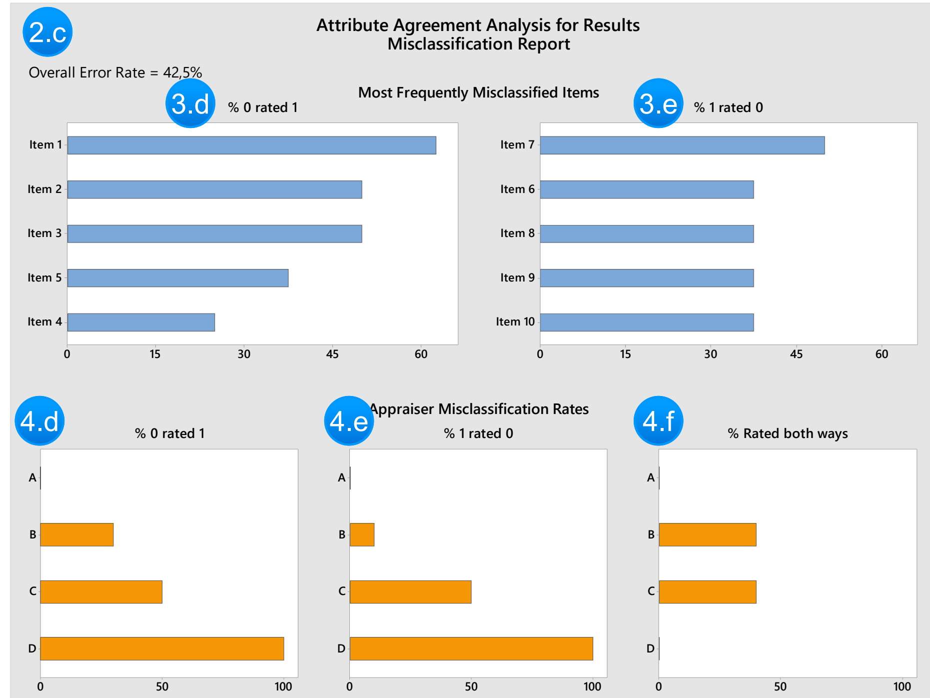
Dialog: Attribute Agreement Analysis	
a.	Column with name of Appraisers (in previously created Worksheet)
b.	Column with trial numbers (in previously created Worksheet)
c.	Column with names of the test items (in previously created Worksheet)
d.	Column with answers/ results of evaluations (entered in previously created Worksheet)
e.	Column with the standard evaluations from experts (in previously created Worksheet)
f.	selection of the coded value which was selected for the category "good"

Example: Evaluate the R&R of good vs. bad Inputs or Outputs (like Decisions or M&M´s)

1.

Attributive Agreement Analysis: Evaluation of Nominal-Scaled Measurements





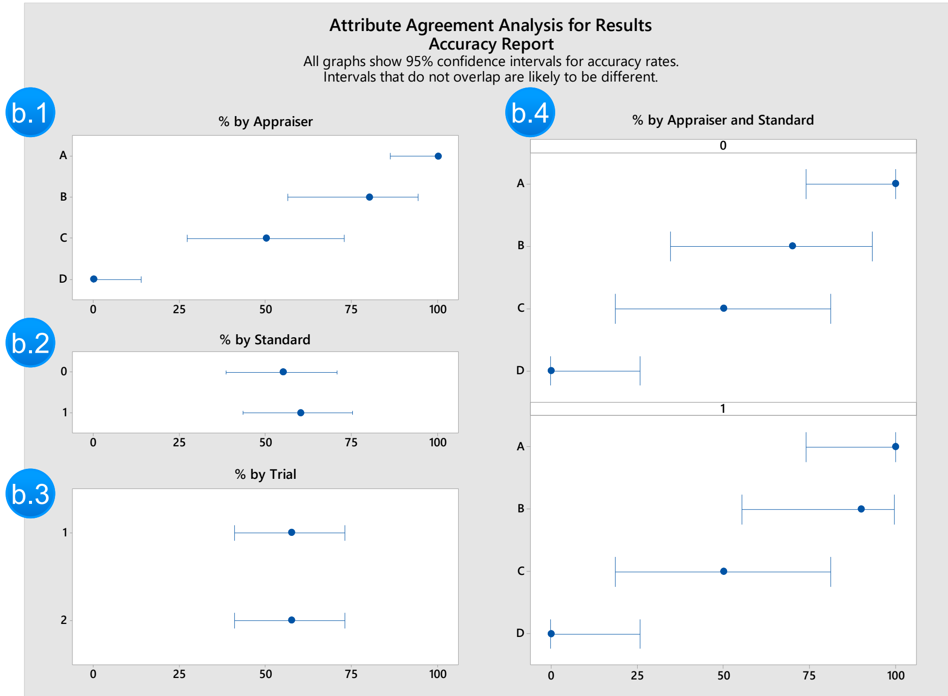
Result: Report Card	
1.	Mix of Items: the highest power to differentiate between appraisers results from a balanced mix of good (0) vs. bad (1) items.
2.	Accuracy and Error Rates: Example for the calculation of:
a.	- Calculation of Number of appraisals overall
e.	- % bad rated good (= false alarms)
d.	- % good rated bad (= missed signals)
	- % good rated good (= hit)
	- % bad rated bad (= correct rejection)
b.	- Overall % Accuracy= % hit + % correct rejection
c.	- Overall % Error Rate= % false alarms + % missed signals
f.	- % Inconsistency (% accurate in one trial and error in another trial)

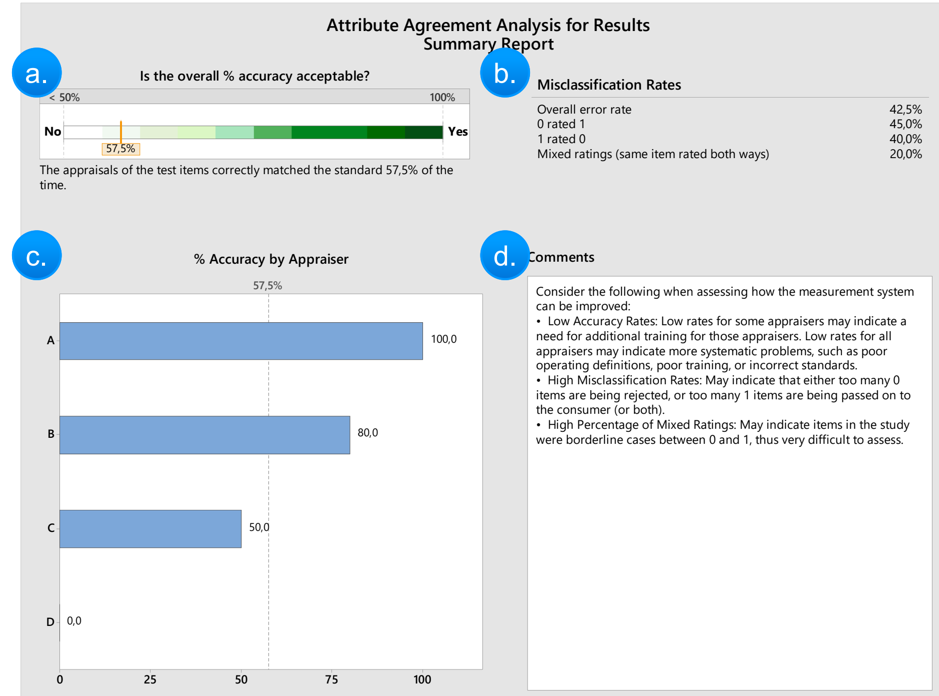
Result: Misclassification Report	
3.d	Evaluation of Items: % good rated bad (= missed signals)
3.e	Evaluation of Items: % bad rated good (= false alarms)
4.d	Evaluation of Appraiser: % good rated bad (= missed signals)
4.e	Evaluation of Appraiser: % bad rated good (= false alarms)
4.f	Evaluation of Appraiser: % Inconsistency (% accurate in one trial and error in another trial)

Example: Evaluate the R&R of good vs. bad Inputs or Outputs (like Decisions or M&M´s)

1.

Attributive Agreement Analysis: Evaluation of Nominal-Scaled Measurements





Result: Accuracy Report	
b.1	Appraiser Accuracy % (= % correct rejections + % hits; with Confidence Intervals (CI)) (Hyp: There is a/ no Difference in: evaluations between: the Appraisers (A vs. B vs. C …))
b.2	Standard Accuracy % (= % correct rejections + % hits; with Confidence Intervals (CI)) (Hyp: There is a/ no Difference in: evaluations between: the two standards (good vs. Bad))
b.3	Trial Accuracy % (= % correct rejections + % hits; with Confidence Intervals (CI)) (Hyp: There is a/ no Difference in: evaluations between: the Trials (1st vs. 2nd vs. …))
b.4	Appraiser by Standard Accuracy % (= % correct rejections + % hits; with CI´s) (Hyp: There is a/ no Difference in: evaluations between: the Appraisers (A vs. B vs. C …) between: Standards (good vs. bad) Look for interactions, i.e. reciprocal results on the two Levels of Standard

Result: Summary Report	
a.	Is the overall % accuracy acceptable? Overall % Accuracy (% hits + % correct rejections) with bar-chart (50% - 100%), indicating the answer (no - yes) with an orange line). (50% = random Accuracy; 100% = perfect Accuracy)
b.	Misclassification Rates: - Overall % Error Rate= % false alarms + % missed signals - % good rated bad (= missed signals) - % bad rated good (= false alarms) - % Inconsistency (% accurate in one trial and error in another trial)
c.	Appraiser Accuracy % (= % correct rejections + % hits)
d.	Comments: Summary and comments about results

Example: Evaluate the R&R of good vs. bad Inputs or Outputs (like Decisions or M&M´s)

Manual Calculation: Attributive Gage R&R …

known Population Sample #	known Population Attribute	Operator 1 Try 1	Operator 1 Try 2	Operator 2 Try 1	Operator 2 Try 2	Operator 3 Try 1	Operator 3 Try 2	Y/N
1	**ok**	ok	ok	ok	ok	ok	ok	**Y**
2	**defect**	defect	defect	defect	defect	defect	defect	**Y**
3	**ok**	ok	defect	ok	ok	ok	ok	**N**
4	**ok**	ok	ok	ok	ok	ok	ok	**Y**
5	**ok**	ok	defect	ok	ok	ok	ok	**N**
6	**ok**	ok	ok	ok	ok	ok	ok	**Y**
7	**ok**	ok	ok	ok	ok	ok	ok	**Y**
8	**defect**	defect	defect	defect	defect	defect	defect	**Y**
9	**defect**	defect	defect	defect	defect	defect	defect	**Y**
10	**ok**	ok	ok	ok	ok	ok	ok	**Y**
11	**ok**	ok	ok	ok	ok	ok	ok	**Y**
12	**defect**	defect	defect	defect	defect	defect	defect	**Y**
13	**ok**	defect	defect	ok	ok	ok	ok	**N**
14	**ok**	ok	ok	ok	ok	ok	ok	**Y**
15	**ok**	ok	ok	ok	ok	ok	ok	**Y**
16	**ok**	ok	ok	ok	ok	ok	ok	**Y**
17	**defect**	defect	defect	defect	defect	defect	defect	**Y**
18	**ok**	ok	ok	ok	ok	ok	ok	**Y**
19	**ok**	ok	ok	ok	ok	ok	ok	**Y**
20	**ok**	ok	ok	ok	ok	ok	ok	**Y**
21	**defect**	defect	defect	defect	defect	defect	defect	**Y**
22	**defect**	defect	defect	defect	defect	defect	defect	**Y**
23	**defect**	defect	defect	defect	defect	defect	defect	**Y**
24	**ok**	ok	ok	ok	ok	ok	ok	**Y**
25	**ok**	defect	defect	defect	defect	defect	defect	**N**
26	**defect**	defect	defect	defect	defect	defect	defect	**Y**
27	**ok**	ok	ok	ok	ok	ok	ok	**Y**
28	**ok**	ok	ok	ok	ok	ok	ok	**Y**
29	**ok**	defect	defect	ok	ok	ok	defect	**N**
30	**ok**	defect	ok	ok	ok	ok	ok	**N**
Appraiser Score:		24/30=	**80%**	29/30=	**97%**	28/30=	**93%**	**80%**
		Repeatability		**Repeatability**		**Repeatability**		**Reproducibility**

… to evaluate the Repeatability and Reproducibility of a Nominal-Scale based Measmt-System

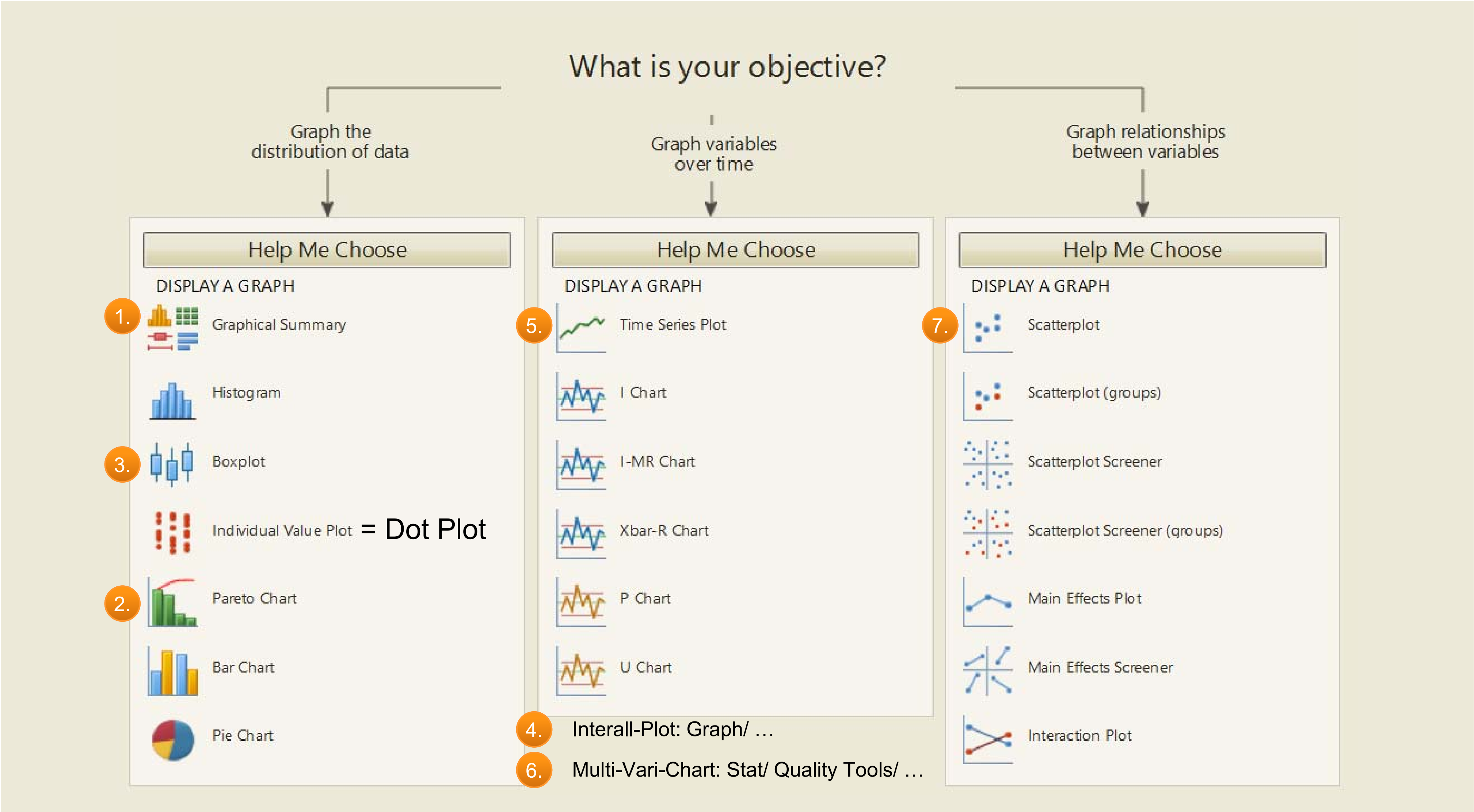
Graphical Analysis

Overview of important Charts …

	Chart	Purpose	Hypothesis	Y	Scale Level	x	Scale Level	Mintab Menu	Options	Alternative
1.	**Histogram**	- Frequency distribution of values for one variable; - Comparison of the data distributions with the ND curve	Difference	1	cardinal			- Assistant/ Graph. Analysis - Chart/ Histogram - Statistics/ Basic Statistics/ Graph. Sum	Grouping	Dot Plot
2.	**Pareto-Diagram**	Distribution of the frequency of results in categories of one variable	Difference	1	nominal			- Assistant/ Graph. Analysis - Statistics/ Quality Tools/ Pareto Chart	Various data formats are accepted	Pie Chart
3.	**Boxplot**	Comparison of the differences between center (median, mean) and dispersion (Quartile) of dependent variable (Y), grouped by an independent variable (x)	Difference	1	ordinal/ cardinal	>=1	nominal	- Assistant/ Graph. Analysis - Graph/ Box Plot (also available in t-Tests/ ANOVA)	Grouping	Interval Plot; Multi-Vari-Chart
4.	**Interval Plot**	Comparison of the difference between averages and confidence intervals of a variable (Y), grouped by an independent variable (x)	Difference	1	cardinal	1	nominal	- Graph/ Interval Plot	Grouping	Boxplot
5.	**Time Series Plot**	Chronological representation of one or more variables	Difference	1	ordinal/ cardinal	1	ordinal/ cardinal time stamps	- Assistant/ Graph. Analysis - Graph/ Time Series Plot	Grouping/ Various Time Stamps	Control Chart
6.	**Multi-Vari Chart**	Compare the differences between the averages of the dependent variable (Y) resulting from multiple scaled independent variables (x)	Difference	1	cardinal	>=2	nominal	- Statistics/ Quality Tools/ Multi-Vari Chart		Box Plot
7.	**Scatter Plot**	Examine the relationship between two variables (Y= f(x))	Relationship	1	cardinal	1	cardinal	- Assistant/ Graph. Analysis - Graph/ Scatter Plot	Grouping/ Regression	Scatterplot (groups)/ Marginal Plot
	Marginal Plot	Examin the relationship between two variables as well as the respective distributions and outliers (Y= f(x))	Relationship	1	cardinal	1	cardinal	- Graph/ Marginal Plot		Scatter Plot
	Matrix Plot	Examine the relationships between multiple variables (multiple Scatter Plots within one chart) (Y= f(x))	Relationship	>=2	cardinal	>=2	cardinal	- Graph/ Matrix Chart	Grouping/ Regression	Multiple Scatter Plots

… to describe and summarize data

Minitab Assistant: Graphical Analysis



I-Chart, I-MR Chart, Xbar-R Chart, P Chart and U Chart are part of Control-Charts

1. Histogram: Frequency Distribution of Variables

Chart

Histogram

Purpose

- Frequency distribution of values for one variable;
- Comparison of the data distributions with the ND curve

Description

x-axis: Categorized values of a variable, e.g. dependent variable (Y)
y-axis: Probability of a variable

Hypothesis

Difference	There are (no) differences in parameters between the distribution of the sample and the ND

Example

Distribution of the body size
Distribution of landing areas for airplanes on the runway

Distribution of the weight of the cookie

Y	Scale Level
1	cardinal
x	Scale Level
./.	

Mintab Menu

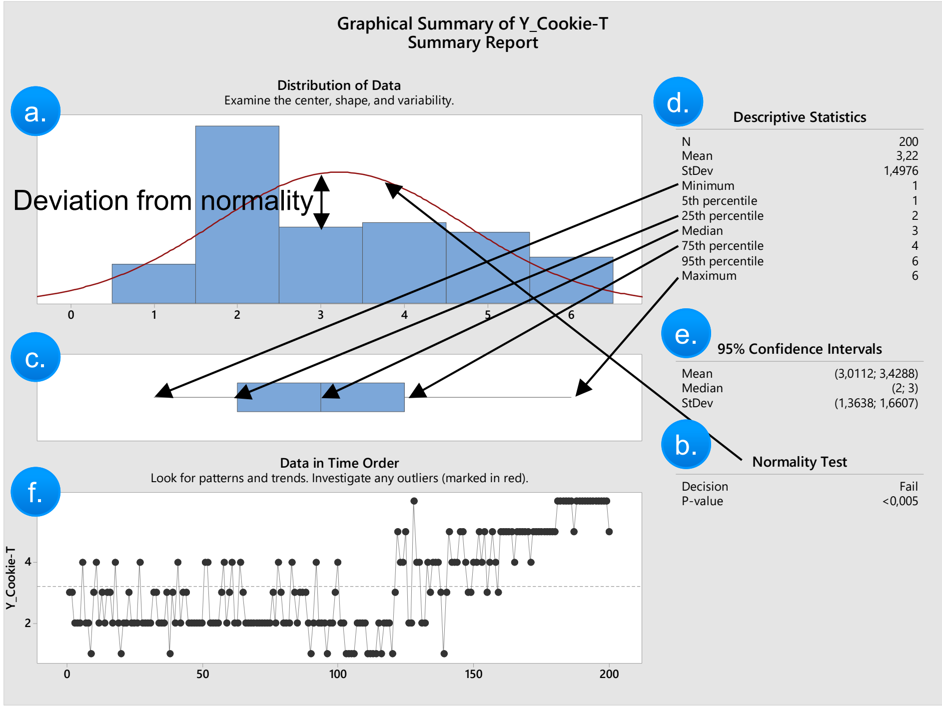
- Assistant/ Graph. Analysis
- Chart/ Histogram
- Statistics/ Basic Statistics/ Graph. Sum

Options

Grouping

Alternative

Dot Plot



a. Histogram of the sample with shape of the expected ND
b. Anderson Darling ND Test: tests if sample and the shape correspond (H0: Sample= ND; HA: Sample <> ND
c. Boxplot with Min; Percentiles (25, 50 (Median), 75) Max
d. Statistics Parameter of the sample
e. Confidence Intervals for the Parameter of the sample as estimations of the Parameter in the population
f. Time Series Plot

Example: Lead Time / Diameter/ Cookie weight/ Scale Level: ordinal, cardinal

2. *Pareto Chart: Freqency distribution of categorized results*

Chart

Pareto Chart

Purpose

Distribution of the frequency of results in categories of one variable

Description

x-axis: Categorized values of a variable, e.g. dependent variable (Y)
y-axis: Probability of a variable

Hypothesis

Difference	There are (no) differences in the parts of categories of variables

Example

Distribution of election results

Distribution of the frequency/ costs of different types of problems

Y	Scale Level
1	nominal
x	Scale Level
./.	

Mintab Menu

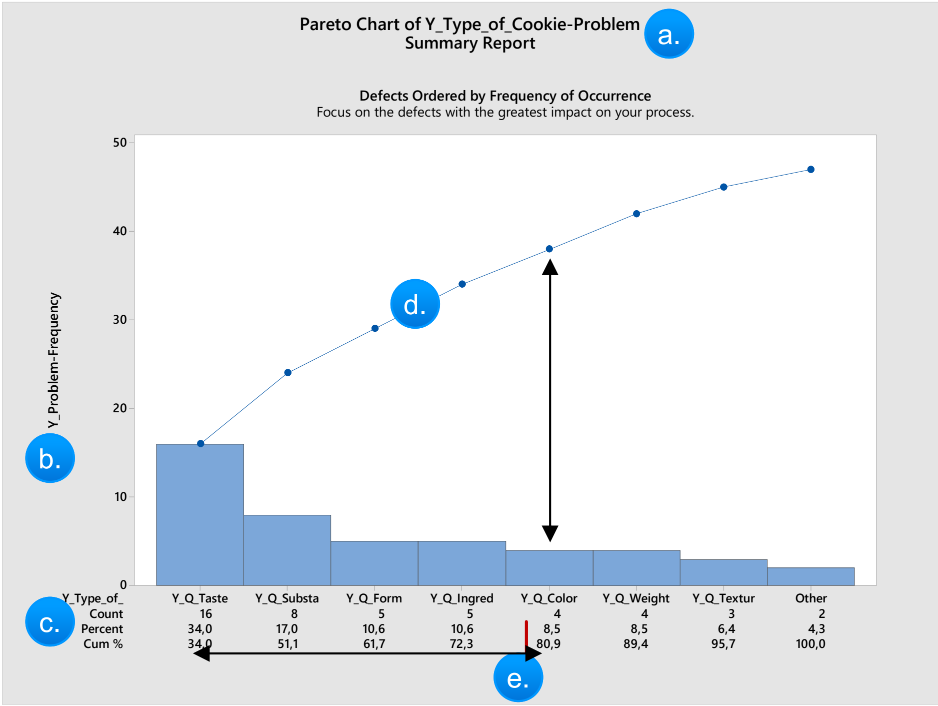
- Assistant/ Graph. Analysis
- Statistics/ Quality Tools/ Pareto Chart

Options

Various data formats are accepted

Alternative

Pie Chart



a. Pareto-Diagram of the distribution of Defects/ Problems
b. Frequency of Deferts/ Problems
c. Defects/ Problems sorted by the frequency (Count/ Percent) plus their cumulative frequency in Percent
d. Cumulative line of frequency
e. Pareto-Principle: 80–20 rule, the law of the vital few, and the principle of factor sparsity. For many deviations roughly 80% of the Problems (Cookie defect) come from 20% of the causes (Problem-Types)

Example: Problem Frequency of Types of defects/ Problems / Scale Level: nominal

3. *Box-Plot: Integrated plot of the center and the dispersion of (grouped) variables*

Chart

Box Plot

Purpose

Comparison of the differences between center (median, mean) and dispersion (Quartile) of dependent variable (Y), grouped by an independent variable (x)

Description

x-axis: nominal scaled categories (Factor levels)
y-axis: dependent variable (e.g. cycle time, errors per order)

Hypothesis

Difference	There are (no) differences in the values of the dependent variable (Y) between the factor levels (x)

Example

Comparison of compensation based on gender and profession

Comparison of the weight of Chocolate vs. Vanilla" cookies

Y	**Scale Level**
1	ordinal/ cardinal
x	**Scale Level**
>=1	nominal

Mintab Menu

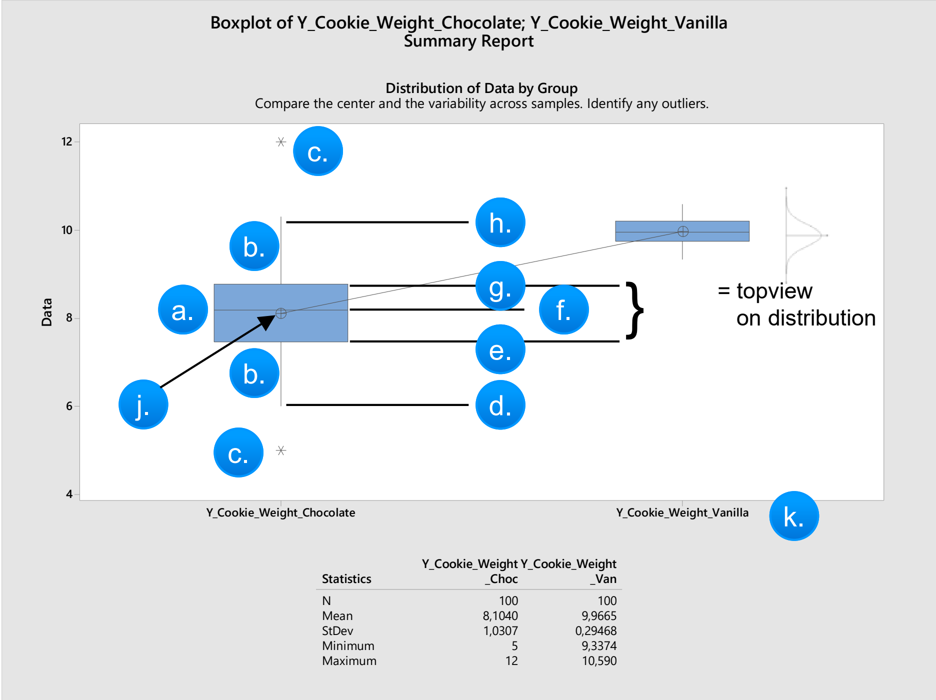
- Assistant/ Graph. Analysis
- Graph/ Box Plot
(also available in t-Tests/ ANOVA)

Options

Grouping

Alternative

Interval Plot;
Multi-Vari-Chart



a. Box
b. Whisker
c. Outlier
d. Smallest not extreme value (Minimum if there are no outliers)
e. 1. Quartile (= Q1: 25% cases below)
f. 2. Quartile (= Q2: 50% of cases below and above = Median)
g. 3. Quartile (= Q3: 75% cases below)
h. Biggest not extreme value(Maximum if there are no outliers)
i. Interquartil-Range (IRQ) = 50%
j. Mean
k. Grouping of Results (Y) by Factors (x)

Example: Cycle Time duration / Cookie Weight/ Cookie Taste / Scale Level: ordinal, cardinal

4. *Interval-Plot: Differences in Parameters of central tendency (Y) between groups (x)*

Chart

Interval Plot

Purpose

Comparison of the difference between averages and confidence intervals of a variable (Y), grouped by an independent variable (x)

Description

x-axis: nominal/ ordinal or cardinal scaled categories (Factor levels)
y-axis: dependent variable (e.g. cycle time, weight)

Hypothesis

Difference	There are (no) differences in the values of the dependent variable (Y) between the factor levels (x)

Example

Area of normal, i.e. expected values for parameters of medicine

Comparison of the consistency of "dark" vs. "light" cookies

Y	**Scale Level**
1	cardinal
x	**Scale Level**
1	nominal

Mintab Menu

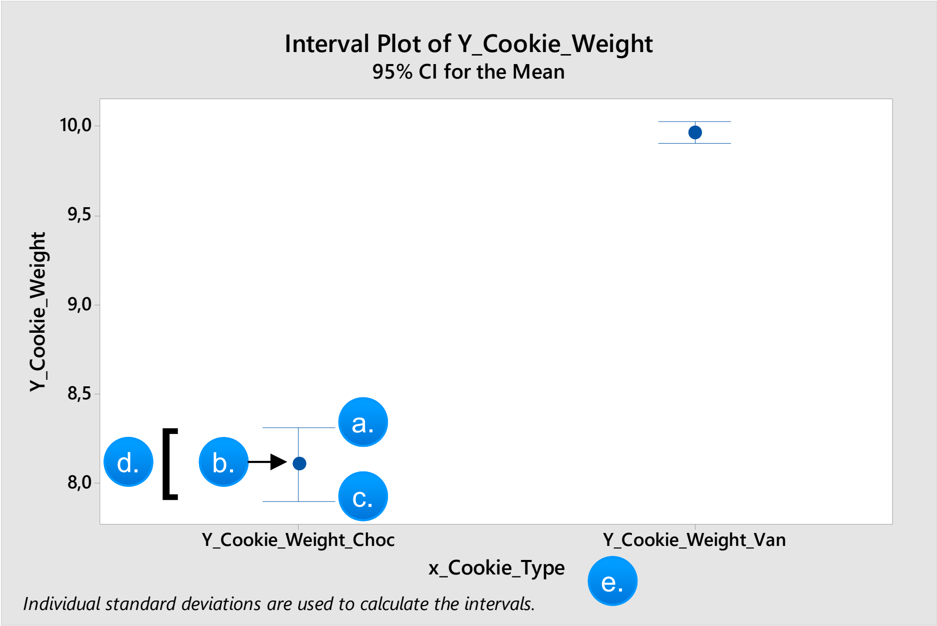
- Graph/ Interval Plot

Options

Grouping

Alternative

Boxplot



a. Upper Confidence Limit for Mean
b. Mean
c. Lower Confidence Limit for Mean
d. Confidence Interval CI (error margin of the Mean)
:= with a 95% Confidence the Mean of the Population is expected within this Interval (CI= xbar +/- 1,96* s/sqrt(N))
e. Grouping of Results (Y) by Factors (x)

Example: Cycle Time duration / Cookie Weight/ Cookie Taste / Scale Level: cardinal

5. *Time Series Plot : Representation of one or more varibales in chronological order*

Chart

Time Series Plot

Purpose

Chronological representation of one or more variables

Description

x-axis: Time Series (e.g. Days/ Hours/ order number) in chronological order
y-axis: dependent variable (e.g. cycle time, errors per order)

Hypothesis

Difference	There are (no) differences in the values of the dependent variable (Y) between the time points (x)

Example

"Temperature Curve"

Progression of the cookie consumption in the month of December

Y	Scale Level
1	ordinal/ cardinal
x	Scale Level
1	ordinal/ cardinal time stamps

Mintab Menu

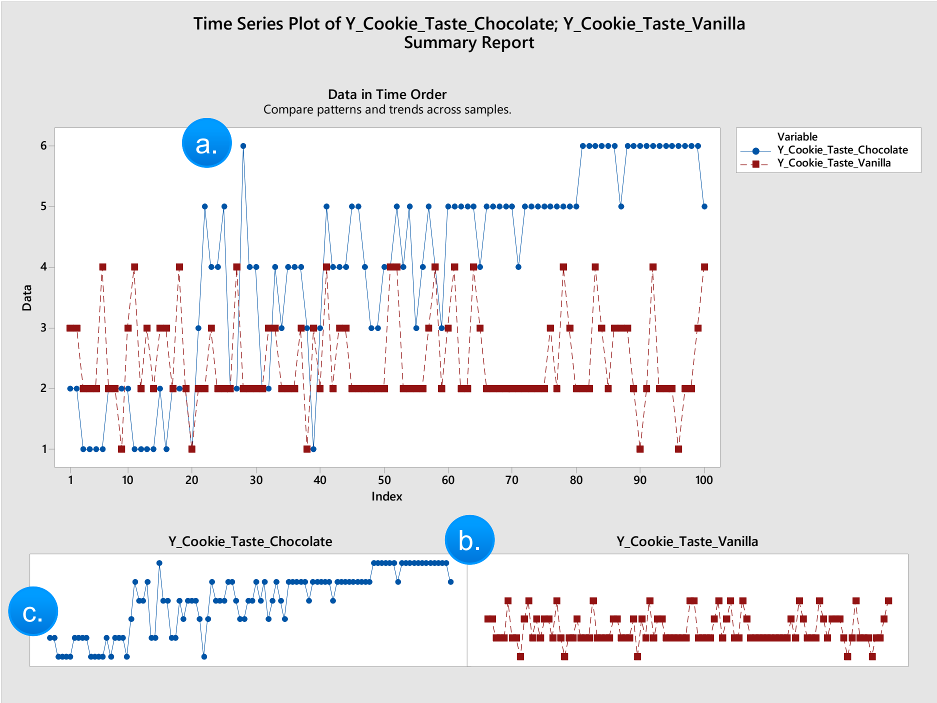
- Assistant/ Graph. Analysis
- Graph/ Time Series Plot

Options

Grouping/ Various Time Stamps

Alternative

Control Chart



a. Every data point represents a measurement of the dependent variable (Y) at a specific point in time or in a chronological series (x)
b. By grouping the parallel progression of two or more interacting variables can be represented
c. After a 90° rotation a time series plot becomes a histogram, if the data points fall on the Y-axis and are stacked
d. The Time Series Plot is the basis for the Control Charts

Example: Lead Time over time/ Cookie Taste over time / Scale Level: cardinal

6. Multi-Vari Chart: Differences in Y between n>1 levels of n> 1 factors x

Grafik

Multi-Vari-Diagramm

Zweck

Vergleich der Unterschiede von Mittelwerten einer Ergebnisvariable (Y) in Abhängigkeit mehrerer gestufter Einflussvariablen (x)

Beschreibung

x-Achse: nominal/ ordinal oder kardinal gestufte Kategorien (Faktor-Stufen)
Y-Achse: Ergebnisvariable (z.B. Durchlaufzeit/ Fehler pro Auftrag)

Hypothese

Unterschied	Es gibt (k)einen Unterschied in: den Werten der Ergebnisvariable (Y) zwischen: den Faktorstufen (x)

Beispiel

Vergleich des Gehalts nach Geschlecht, Beruf, Region, Land, Altersstufe

Vergleich des Geschmacks (Y) verschiedener Kekstypen, die aus unterschiedlichen Merkmalskombinationen resultieren (x)

Y	Skalen-Niveau
1	kardinal
x	Skalen-Niveau
viele	kardinal

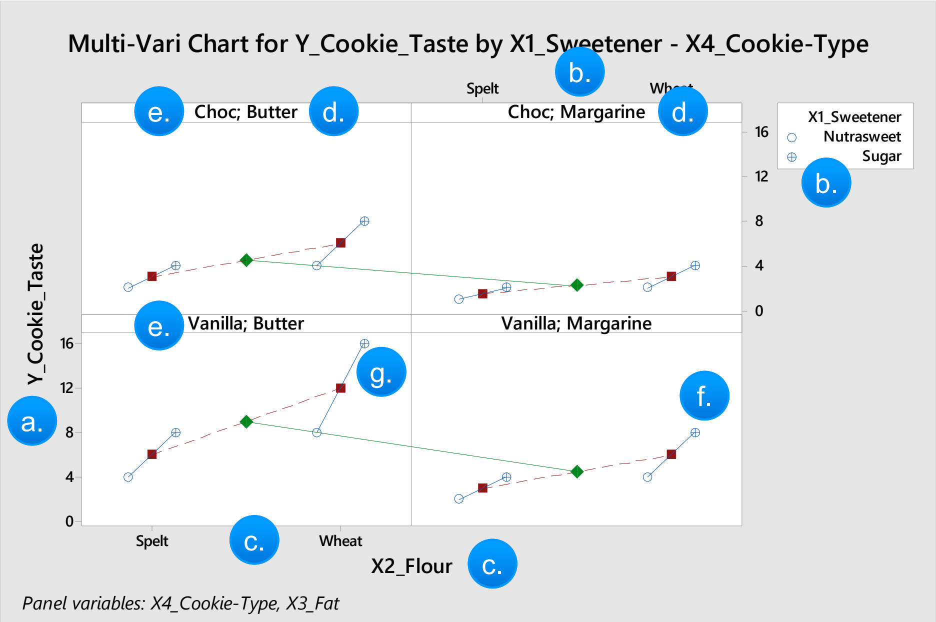
Mintab Menü Aufruf

- Statistik/ Qualitätswerkzeuge/ Multi-Vari-Bild

Optionen

Alternative

Box-Plot



a. Differences in the dependent variable (Y) resulting from multiple independent factors (x)
b. X1: type of sweetener (Spelt vs Wheat)
c. X2: type of flour (Spelt vs Wheat)
d. X3: type of fat (Butter vs. Margarine)
e. X4: type of Cookie (Chocolate. vs. Vanilla)
f. Every data point represents the Mean of the Y-values for a specific combination of levels of multiple x.
g. The greater the slope between two data points, the greater influence of the compared factor levels
h. The Multi-Vari Chart is an alternative graphical representation for the results of the ANOVA

Example: Screening of complex influence factors (x) on the Problem (Y)/ Scale Level: cardinal

7. *Scatter Plot: representation of the Relationship between two or more variables*

Chart

Scatter Plot

Purpose

Examine the relationship between two variables

Description

x-axis: continuous independent variable (x)
y-axis: continous dependent variable (e.g. cycle time, error per job)

Hypothesis

Relationship	There is (no) ("the...the...") relationships between: the independent variable (x) and the dependent variable (Y)

Example

Relationship between: gasoline consumption vs. speed

Relationship between: baking time of the dough (x) and the brightness of the cookie (Y)

Y	Scale Level
1	cardinal
x	Scale Level
1	cardinal

Mintab Menu

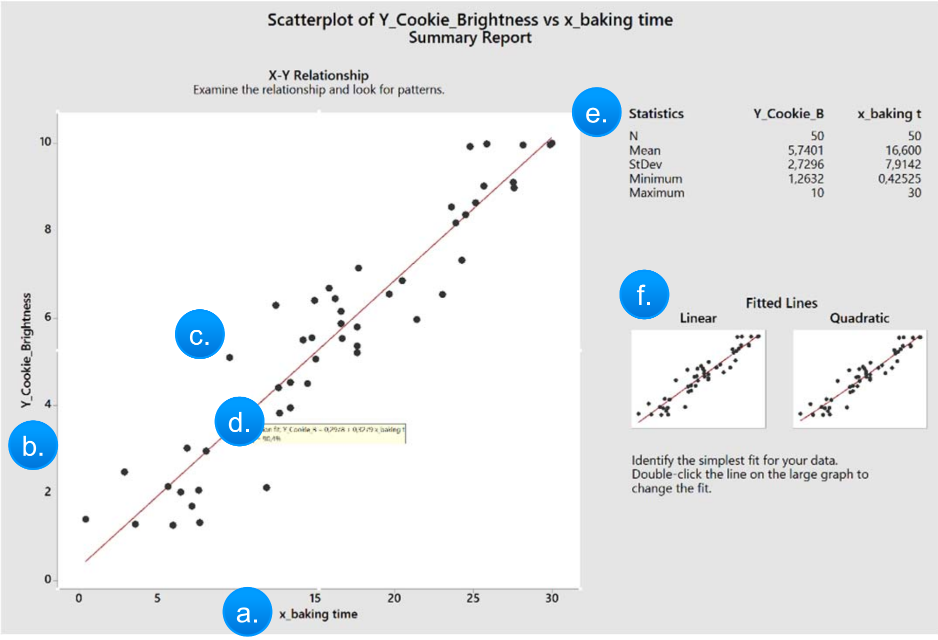
- Assistant/ Graph. Analysis
- Graph/ Scatter Plot

Options

Grouping/ Regression

Alternative

Scatterplot (groups)/ Marginal Plot



a. X-Variable: Length of baking time (from 1- 30 min)
b. Y-Variable: Brightness of Cookies (from bright=0 to dark=10)
c. Data points for each measured Cookie
d. (Linear) Regression line: Y= b+ ax (+error)
e. Descriptive Statistics/ Parameter
f. Type of relationship (Linear, Quadratic, …)

The scatter plot is the graphical display for the:
- Correlation (r_{xy}) and the
- Regression Analysis (Y= b+ ax +e)

Example: Relationship Y= f(x) between speed (x) and mileage (Y)/ Scale Level: Cardinal

Process Capability and Process Control

Introduction

Process control refers to the historical compliance with calculated control limits

Process Control:

- Focuses on single, obvious, systematic, observations over time
- Sets the sequence of the results in relation to the control limits (**for example: X+/- 3s)**
- The control limits are calculated using the variation of the results and therefore mirror past results
- The following becomes relevant depending on the extent of the variation and range of values :
  - Upper control limit (UCL)
  - Lower control limit (LCL)
- Through the control limits and other tests more systematic influences are identified:
  - **Outliers**
  - **Patterns** and
  - **Trends**.
- **Tool** : Control Chart (xbar/s, …)

Process Capability :

- Considers the position **(e.g. Mean)** and the scatter (**z.B. Standard Deviation)** of the random variation in the results
- Puts position and scatter of the results in relationship to the specification limits and/ or a target value (:= Target)
- Specification limits are based on the requirements from „outside“ e.g. customers
- Depending on requirements and the range the following become relevant:
  - Upper specification limit (USL)
  - Lower specification limit (LSL)
  - Target value
- The level of the process capability reveals :
  - the size of the deviation of the results from the specification limits and
  - how accurate the target values meet the goal
- **Tool**: Process Capability Index (cp/ cpk, dpu, dpmo, Sigma-Level, …)

Process Capability refers to the compliance with externally defined specification limits

The Type of Variation Defines the Type of Problem

Systematic variation:

- Spars and few deviations in the results of the dependent variable (**Y**)

Systematic influences (x), are

- Occurs rarely,
- Mostly as a result of having few and
- typically easier to identify causes

Random variation:

- Across all data points and permanent variation in the dependent variable (**Y**)

Random influences (x), are

- latent and permanent occurance,
- is mostly as a result of having many and
- harder to identify the cause

The Type of Problem Determines the Causal Analysis

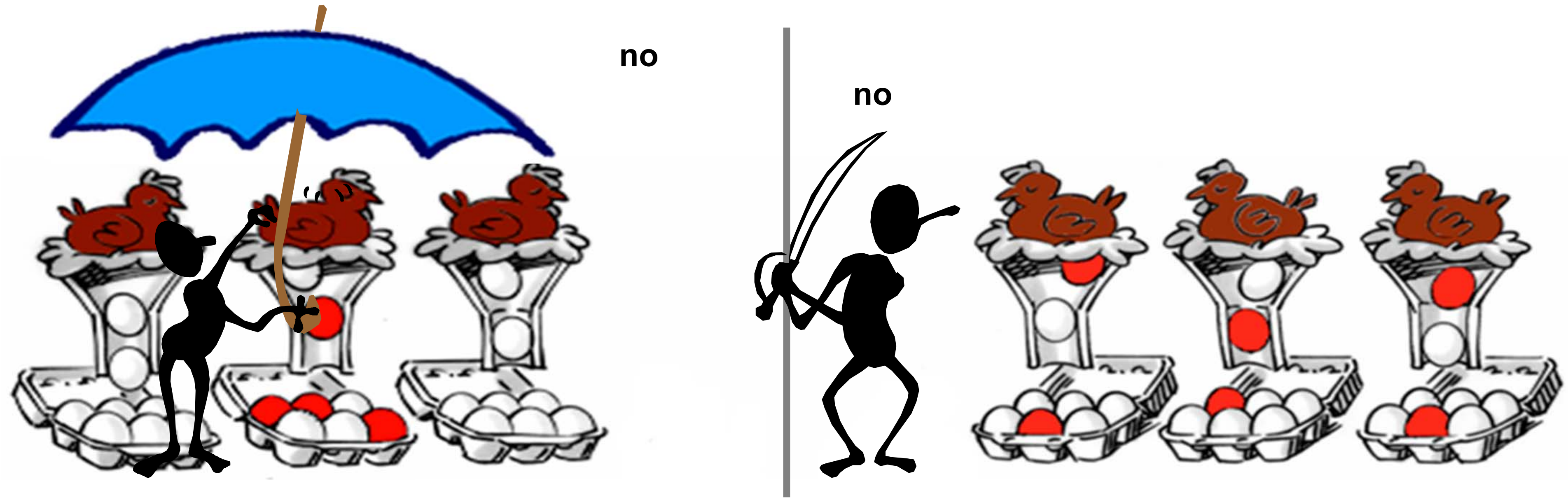
Effective process optimization is only possible if …

If you treat a systematic variation like a random variation you loose the opportunity to…

- to eliminate a permanent and specific **negative** causes
- to use a specific positive trigger as an information of best practice for improvement

When you treat a random variation like a systematic variation…

- you will most likely increase the variation by repeated adjustments of the process parameter



… the type of variation - systematical vs. random – is identified

Strategies for systematic and random variations

Systematic Variation:

- Continuously gather the needed data so that the deviations can be quickly identified
- Find the cause of each deviation. Find why or how that cause came about e.g. what was different in this situation
- Find in the causes a lever to improve the situation
- Implement an improvement as soon as possible to avoid damages and eliminate future outliers
- Develop a longterm provision that eliminates the possibility of repetition of the systematic error

On the other hand:

- With positive outliers the cause can help to systematically improve the process

Random Variation:

- The target is to improve a stable process
- Do not try to find causes for single events, just like in systematic variation
- Variation is seldom reduced by identifying causes for differences between two data points because all data points relevant
- The optimization requires basic changes to the processes
- The sample must be investigated for relationships between independent and dependent variables: $Y= f(x_I, x_M, x_R)$
- Hypotheses must be formed and data must be statistically

The types of causes of variation define the improvement strategies

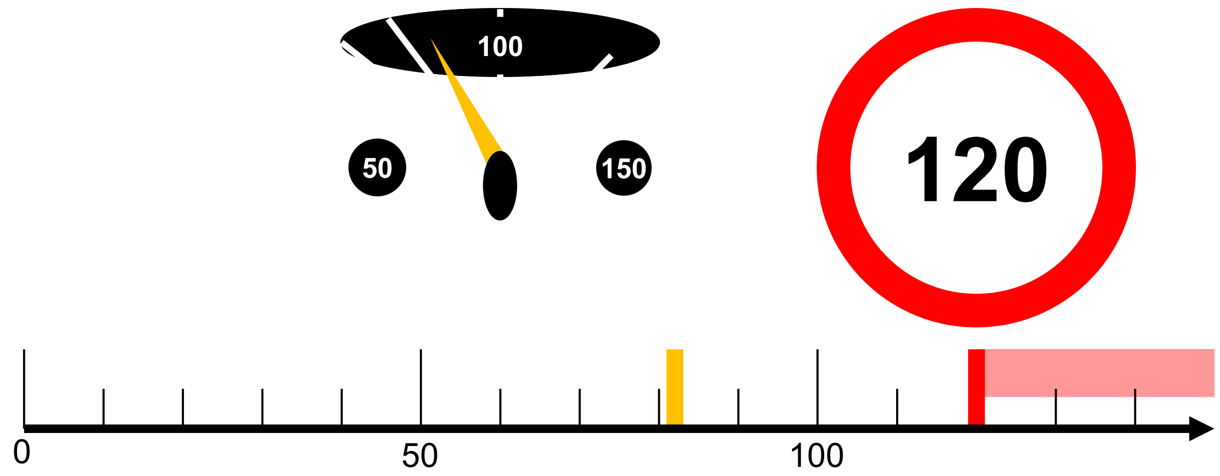
Process Capability

Details

Examples of specification limits can be found in many areas of life

From every day life we already have an idea of the specification limit spectrum, for example, the standard „quarter of an hour“ with appointments, the cut off points with grades (must be better than Grade x), the minimum requirement for elections (more than x%), the speed limit on a street and the experation date of a product.

The specification limit defines a value on a scale that defines acceptable and not acceptable, basically it separates problematic and not problematic result areas. Through the comparison of the product‘s results with its specification limits it can be deduced if the product meets expectations.



The process capability is deduced from the comparison of the observed values with the specified expectations. This comparison comes from the calculation of a process capability measurement.

The specification values separate the acceptable from the unacceptable values

The suitable process capability analysis depends upon...

Determination of Scale Level:

1. A basic evaluation of product quality is possible through a measurement of the product as a whole:
   {**ok** vs. **ko**}, {**works** vs. **does not work**}, {**yes** vs. **no**} oder {**right** vs. **wrong**}.
   Here are identified two classes and through the measurements the product is categorized to one. The measurements here are **attributive** on a **nominal scale**.
   The specification limits and the target are defined through the acceptable percentage of errors.
2. An advanced evaluation of a product‘s characteristic is possible, if the characteristic can be measured on a continuous scale, e.g. one an ordinal- or a Cardinal Ccale:
   {**km/h**}, {**Time/ Product**} oder {**Volume/ Time**}.
   The specification limts and the target are identified as values on the scale.

Scale Level of the data of the relevant Product/ Service attribute (CtQ)			
Nominal		**Ordinal/ Cardinal**	
acceptable	**problematic**	**acceptable**	**problematic**
below USL	above USL	above LSL	below OSG
		below USL	above LSL
		between LSL and USL	outside the LSL - USL interval

… the Scale Level of the product characteristics and the distribution of the measurements

Criteria and characteristics of important Process capability Analyses

Process-Performance Tool	**Error-%** (1- Yield-%)	**Error/ Unit** (DPU)	**cpk/ ppk/ Z-Bench**
Example	Defective cookies/ baking sheet Routes with traffic jam/ all routes	Defective/ Cookie Incorrect transfers/ Call Traffic jams/ Route	Weight of the cookies within specification limits Call times within specification limits Delivery accuracy within specification limits
Distribution	Binomial	Poisson	Normal
Objective			
USL	Tolerated Error-%	Tolerated Defects/ Unit	Tolerated value of the upper limit on the scale
LSL	./.	./.	Tolerated value of the lower limit on the scale
Target Values	./.	./.	Target value on the scale
Calculation			
Basis	all products (variable or fixed)	Opportunities * Units	Cp/ Pp= (USL-LSL)/ 6 s
Portion of	defective Products/ Services	Defects/ Unit	Cpk/ Ppk= min((USL-Xbar); (Xbar-LSL))/ 3 s
Reference to the formation of Subgroups (subsamples)	Accurate Units, e.g.: - per Hour - per Day - per Shift - per Lot	Accurate Units, e.g.: - concrete product	Accurate Units, e.g.: - per Hour - per Day - per Shift - per Lot
Prerequisite	Stable Process: - no outliers - no trends; no patterns Test with p-Chart	Stable Process: - no outliers - no trends; no patterns Test with with u-Chart	Stable Process: - no outliers - no trends; no patterns Test with with Xbar/s-Chart
Performance-Measure	Comparison with the target Sigma-Level (Z-Bench) Error-%; Yield-%	Comparison with the target DPU-% Error-%; Yield-%	Comparison with the target Cp/ Pp; Cpk/ Ppk Sigma-level (Z-Bench)

The Process capability can be calculated for all Products and Services

Selecting the suitable Process-Capability Analysis …

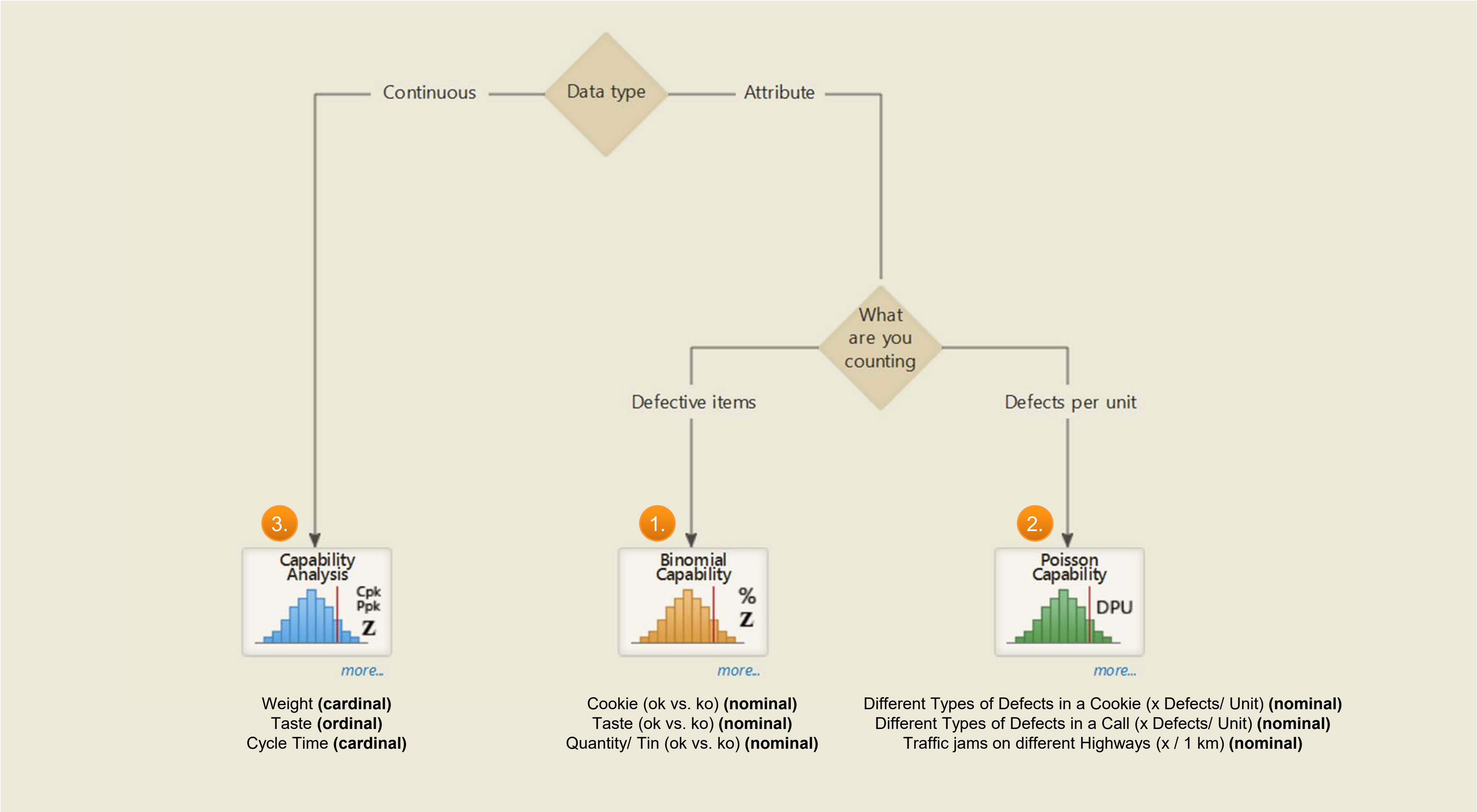
Critical Customer and Business Requirements of the Product: Cookie

Product	Evaluation Category	Characteristic	Target Outcome (ok)				Problem (ko)	Scale Level	Process Capability Analysis		Stakeholder
			LSL			USL			Version 1	Version 2	
Cookie	**Quality**	Weight	18g	Goal: 20g		22g	> 22g	Cardinal/ Ordinal	normal (cp/pp/ Z)	binomial (%-Z)	Customer
		Consistency	Chewy			Crunchy	doughy// crumbly	Ordinal	normal (cp/pp/ Z)	binomial (%-Z)	Customer
		Shape	52 prongs				≠ 52 prongs	Nominal	binomial (%-Z)		Customer
		Sweetness	low	middle		high	neutral, extreme// Salty	Nominal/ Ordinal	binomial (%-Z)	normal (cp/pp/ Z)	Customer
		Colour		sandy beige	chocolate brown		white/ blue/ …	Nominal	binomial (%-Z)		Customer
		Ingredients	1. Vanilla// 2. Coconut// 3. Honey// 4. Lemon// 5. Orange// 6. Almonds// 7. Nuts// 8. Cinnamon// 9. Clove// 10. Nutmeg				missing ingredients	Nominal	Poisson (DPU)	binomial (%-Z)	Customer
		Quantity/ Package	30				< 30	Nominal	Poisson (DPU)	binomial (%-Z)/ normal (cp/pp/ Z)	Customer
	Availability	Quantity	10 Cookies			30 Cookies	0	Cardinal	normal (cp/pp/ Z)		Customer
		Time	24.12./ 14:00			24.12./ 15:30	24.12./ > 15:30	Cardinal	normal (cp/pp/ Z)		Customer
	Used Resources	Price	yi,- €/ Cookie			yj,- €/ Cookie		Cardinal	normal (cp/pp/ Z)		Customer
		Material consumption				4 Eggs	> 4 Eggs	Cardinal	normal (cp/pp/ Z)		Business
		Energy consumption				3 kw	> 3 kW	Cardinal	normal (cp/pp/ Z)		Business
		Working hours				2h	> 2h	Cardinal	normal (cp/pp/ Z)		Business

- The weight of the Cookie can be measured with a scale using a Cardinal Ccale. Then if the weight< 5g (**ko**), 5 < Weight < 20g (**ok**), Weight > 20g (**ko**). The suitable Process Capability Analysis for normal distributed data is: **(normal) cp/pp/ Z**
- The sweetness can attributively be measured on a nominal scale, with : {low, medium, high = **ok** vs. neutral, salty, bitter = **ko**}. The suitable Process Capability Analysis is: **binomial (%-Z)**
- The sweetness can also be measured by a rating on an Ordinal Scale, then for example z.B.: 0= neutral taste (**ko**), 1= low, 2= medium, 3= high sweetness (**ok**), 4= extreme sweet (**ko**). Process Capability for normal distributed data is: **normal (cp/pp/ Z)**
- If 10 ingredients belong to the composition of the Cookie and each missing ingrediant is a Problem (Opportunity for Defect), then the Defects would be the missing ingrediants per Cookie: {0 = **ok** vs. 1-10 = **ko**}. The suitable Process Capability Analysis is: **Poisson (DPU)**

… based on the Scale Level of the tolerated characteristics and the definition of the Problem

The selection of the suitable Process Capability Analysis



Processes are affected by many influences – the Outputs can be checked by various analyses

1.

Binomial Process Capability Analysis (1/3)

Process Capability Analysis

Binomial Capability

Purpose

Checks whether a certain Process is capable to yield Products/ Services that meet Business/ Customer Requirements.

Focus

Overall status of a Product/ Service as being: defective vs. not defective

Evaluation

Chance (p) that a selected Product/ Service is defective. The data collected are the number of defective Products in individual subgroups, which is assumed to follow a binomial distribution with parameter p.

Data

- Number of defects
- in a series of constant/ variable subgroups of a sample

Specification Limits

Upper Specification Limit (USL)

Result

- % defective and its 95% Confidence Interval (CI)
- ppm/ dpmo
- Process Z (Sigma-Level)

Probability Mass Function

n= Number of Products
k= Number of Defects
p= prob(Defect)

$$f(k;n,p) = \Pr(X = k) = \binom{n}{k} p^k (1-p)^{n-k}$$

for $k = 0, 1, 2, ..., n$, where

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

Example

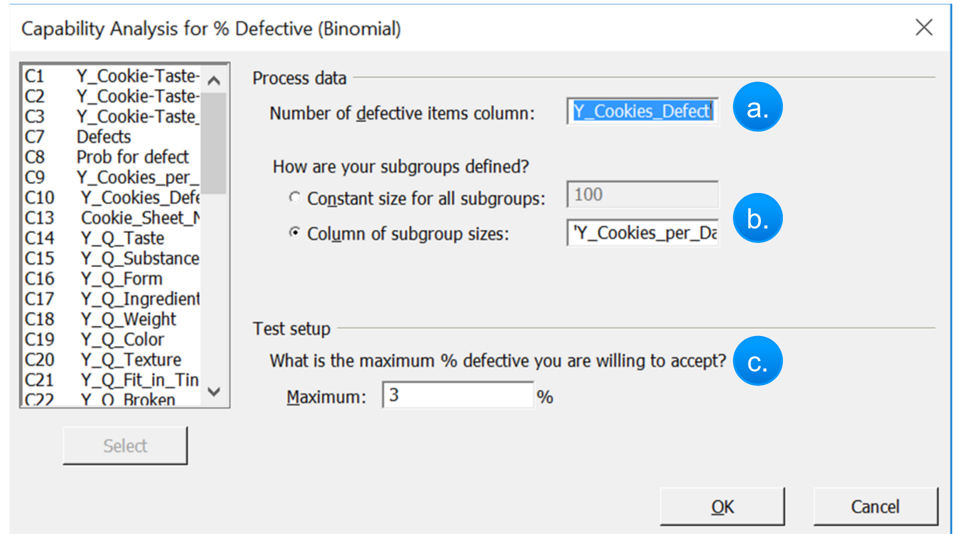
engine starts (yes/ no)
mobile network access (yes/ no)

Cookie (ok/ ko)

Y	Scale Level
1	nominal
	Transform Scale Values
	not defective = 0 vs. defective= 1

Mintab Menu

- Assistant/ Capability Analysis/ Binomial Capability
- Stat/ Quality Tools/ Capability Analysis/ Binomial



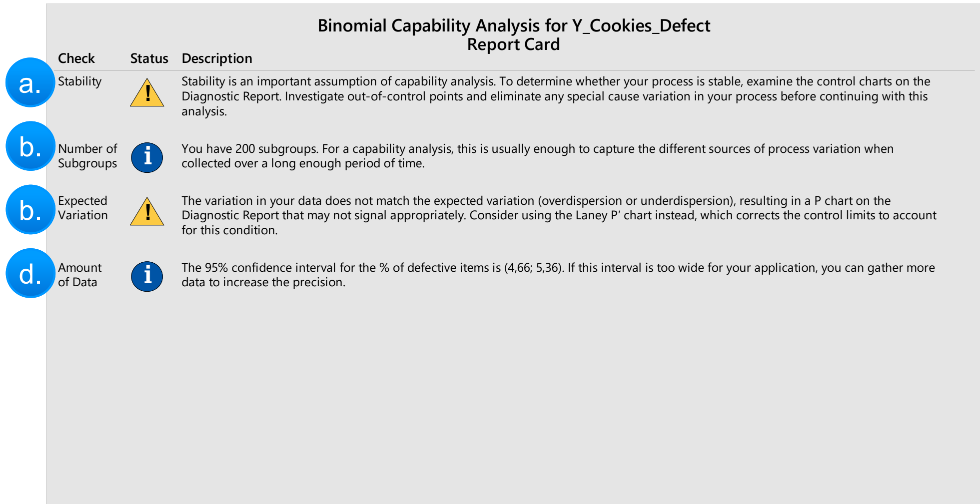
Dialog

a.	Y-Variable with the summarized number of defects in a subgroup (defects/ day; defects/ batch; defects/ shift; defects/ sheet)
b.	1. Constant Size for all Subgroups: e.g. always 30 Cookies/ sheet (which is related to the number of defects) 2. Column of Subgroup Sizes: e.g. variable number of Cookies/ day (which is related to the number of defects)
c.	Upper Specification Limit (USL) in % (Example: 3%)

Capability for the overall statement: Product/ Service is: ok vs. ko

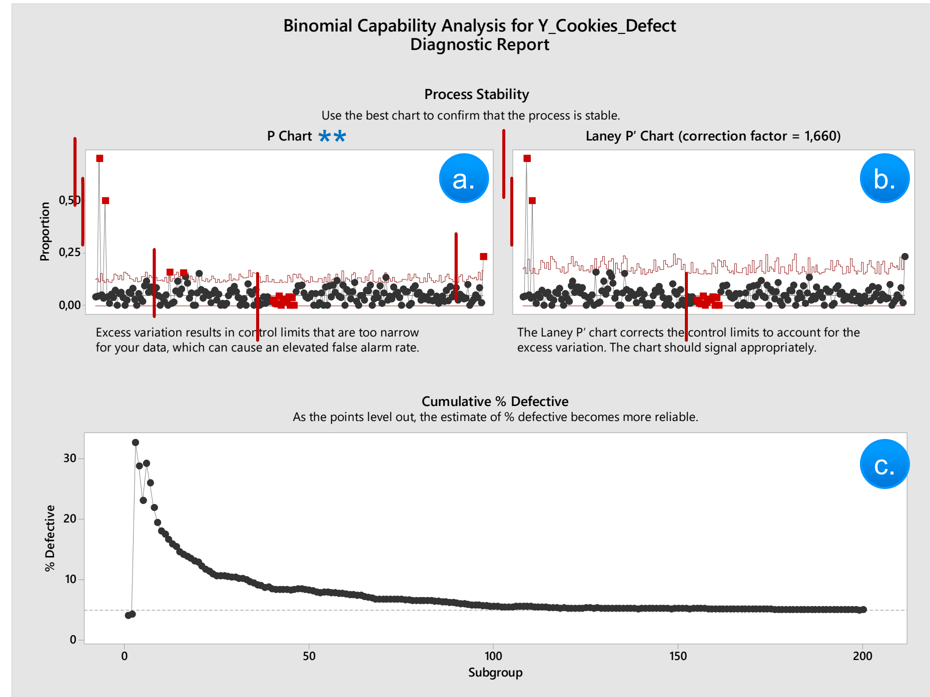
1.

Binomial Process Capability Analysis (2/3)



Binomial Capability Analysis for Y_Cookies_Defect
Report Card

Check	Status	Description
a. Stability	!	Stability is an important assumption of capability analysis. To determine whether your process is stable, examine the control charts on the Diagnostic Report. Investigate out-of-control points and eliminate any special cause variation in your process before continuing with this analysis.
b. Number of Subgroups	i	You have 200 subgroups. For a capability analysis, this is usually enough to capture the different sources of process variation when collected over a long enough period of time.
b. Expected Variation	!	The variation in your data does not match the expected variation (overdispersion or underdispersion), resulting in a P chart on the Diagnostic Report that may not signal appropriately. Consider using the Laney P' chart instead, which corrects the control limits to account for this condition.
d. Amount of Data	i	The 95% confidence interval for the % of defective items is (4,66; 5,36). If this interval is too wide for your application, you can gather more data to increase the precision.



Report Chart with statements about:

a. Stability of the Process:
- Outliers and Patterns in the data indicate low Stability of the Process
> interpret Parameters of Process Capability with reservation

b. Number of Subgroups:
- too few observations (subgroups) indicate short term observation
> interpret Parameters of Process Capability with reservation

c. Expected Variation: Overdispersion and Underdispersion
- Variation in Subgroup Sizes can lead to Overdispersion and false alarms
- Intercorrelation of Subgroup results can lead to Underdispersion and missed signals

d. Amount of data:
- too few data/ small sample size might prevent Significance
> Collect more data

Control Charts to asses the stability of the Process

a. p-Chart for the Process
= Time Series Plot with Upper/ Lower Control Limits (UCL/ LCL)
= Outliers and Patterns indicate deviation from Stability

b. Laney p-Chart for the Process
= corrected p-Chart due to overdispersion
= Outliers and Patterns indicate deviation from Stability

c. Chart with cumulative % Defective:
- an asymptotic progress of the line indicates the reliability of the estimation of the % Defective Process Capability Parameter

Check fulfillment of prerequisite conditions

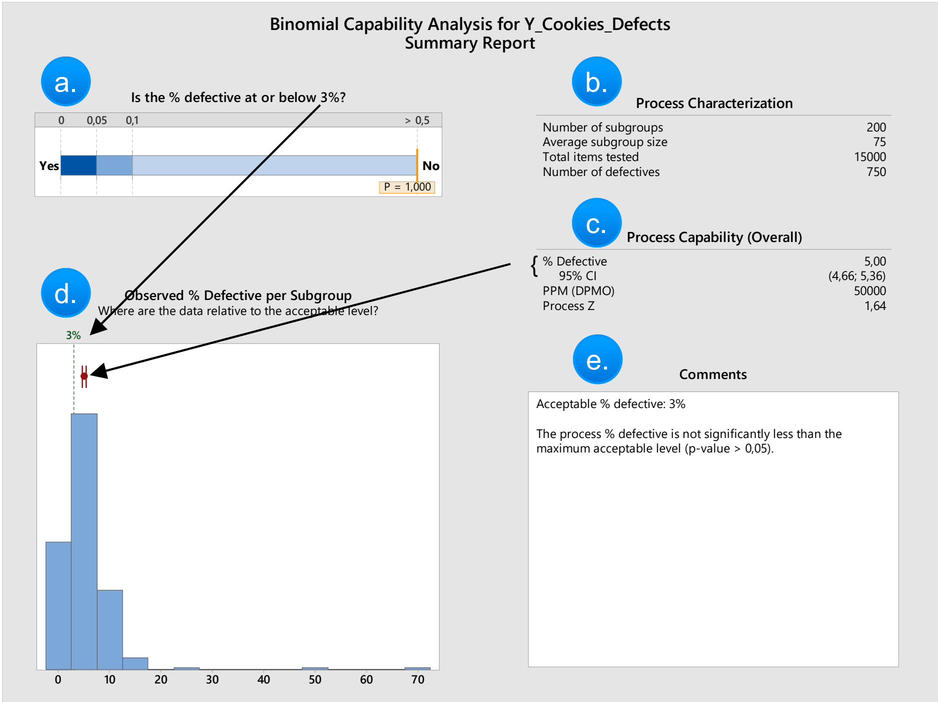
** See also Control Charts: p-Chart

1.

Binomial Process Capability Analysis (3/3)

Results

a.	Significance Test (here: 1p-Test, One-Sample-p-Test): H0: % Defective > USL (Example: 3%) HA: % Defective <= USL (Example: 3%) The bar of the chart indicates: - orange line: actual p-value of the Significance Test - dark blue sector: 0% > alpha <= 5% - light blue sector: 5% > alpha <= 10% Example: - p= 1,000 - Confirmation of H0 - % Defective is not <= 3% (USL)
b.	Process Characterization: Description of the Sample: - Number of subgroups (Nos) - Average subgroup size (Ass) - Total items tested (= Nos * Ass) Result: - Number of defectives
c.	Process Capability: - % Defective: 5% (= 750/ 15.000) - 95% CI (for % Defective) (4,66 - 5,36) - PPM (DPMO)= 50.000 (/ 1Mio) - Process Z= 1,64 (= Sigma-Level)



d.	Observed % Defective per Subgroup - Bar Chart with the distribution of % Defectives/ Subgroup - green dashed line: USL (3%) - red dot: % Defective - red interval: 95% Confidence Interval (CI) for % Defective Interpretation: - USL is below 95% CI of % Defective -> indicates confirmation of H0
e.	Comment: - Summary of results and additional hints

Results: % defective, ppm/ dpmo, Z (Sigma-Level)

2.

Poisson Process Capability Analysis (1/3)

Process Capability Analysis

Poisson Capability

Purpose

Checks whether a certain Process is capable to yield Products/ Services that meet Business/ Customer Requirements.

Focus

Product/ Service can have multiple defects and the number of defects on each item is counted.

Evaluation

Number of defects per unit. The data collected are the total number of defects in k units contained in individual subgroups, which is assumed to follow a Poisson distribution with an unknown mean number of defects per unit (u).

Data

- Number of defects in a Number of units * opportunity for defect
- in a series of constant/ variable subgroups of a sample

Specification Limits

Upper Specification Limit (USL)

Result

- Defects per Unit (DPU and its 95% Confidence Interval (CI)
- Yield (Probability of producing a unit without defects)

Probability Mass Function

lambda= average number of events per interval
e= 2.71828 (Euler's number, base of natural logarithm)
k= Number of defects
k! = factorial of k

$$P(k \text{ events in interval}) = \frac{\lambda^k e^{-\lambda}}{k!}$$

Example

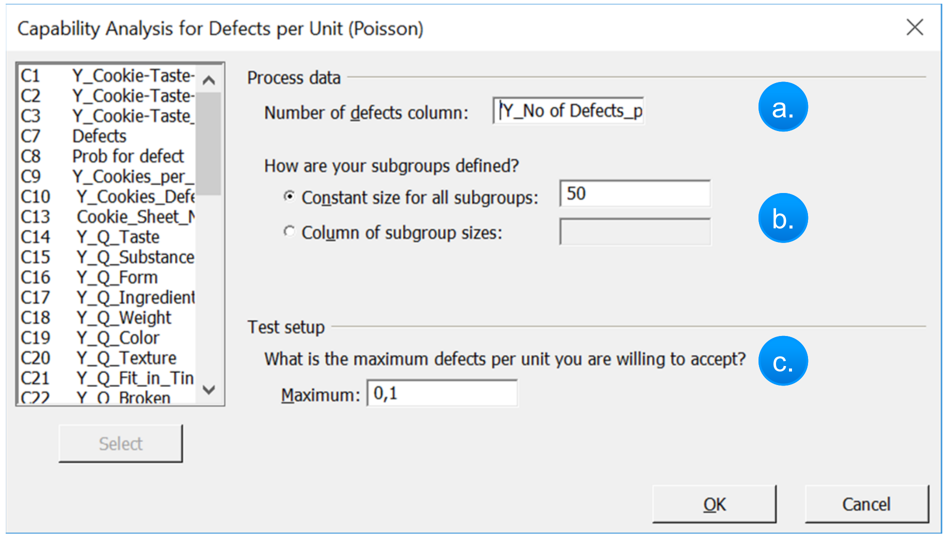
aircraft (1.000.000 Opportunities for Defect) -> 1 aircraft= 1.000.000 units, e.g. compared with Chewing Gum (5 Opportunities for Defect) -> 1 Chewing Gum= 5 units

Cookie (10 Opportunities for Defect) -> 1 Cookie= 10 units

Y	Scale Level
1	nominal
	Transform Scale Values
	Opportunity not defective = 0 vs. defective= 1, count number of defect Opportunities

Mintab Menu

- Assistant/ Capability Analysis/ Poisson Capability
- Stat/ Quality Tools/ Capability Analysis/ Poisson



Dialog

a. Y-Variable with the summarized number of defects in a subgroup (Opportunities * Defects/ sheet)

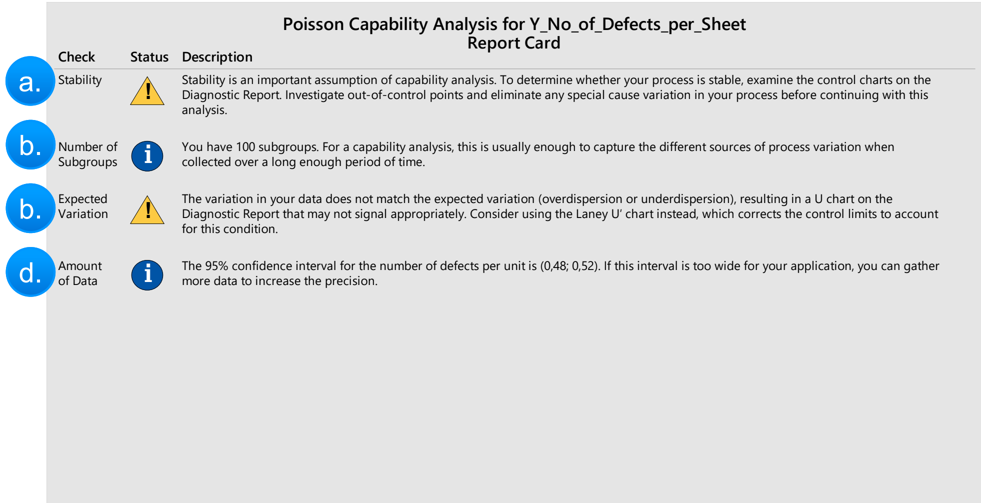
b. 1. Constant Size for all Subgroups: e.g. always 50 Cookies/ sheet (which is related to the number of defects)
2. Column of Subgroup Sizes: e.g. variable number of Cookies/ sheet (which is related to the number of defects)

c. Upper Specification Limit (USL) in DPU
(Example: 0,1)

Capability for the overall statement: Unit (Opportunity * No. of Product/ Service) is: ok vs. ko

2.

Poisson Process Capability Analysis (2/3)


Poisson Capability Analysis for Y_No_of_Defects_per_Sheet
Report Card

Check	Status	Description
a. Stability	⚠	Stability is an important assumption of capability analysis. To determine whether your process is stable, examine the control charts on the Diagnostic Report. Investigate out-of-control points and eliminate any special cause variation in your process before continuing with this analysis.
b. Number of Subgroups	i	You have 100 subgroups. For a capability analysis, this is usually enough to capture the different sources of process variation when collected over a long enough period of time.
b. Expected Variation	⚠	The variation in your data does not match the expected variation (overdispersion or underdispersion), resulting in a U chart on the Diagnostic Report that may not signal appropriately. Consider using the Laney U' chart instead, which corrects the control limits to account for this condition.
d. Amount of Data	i	The 95% confidence interval for the number of defects per unit is (0,48; 0,52). If this interval is too wide for your application, you can gather more data to increase the precision.

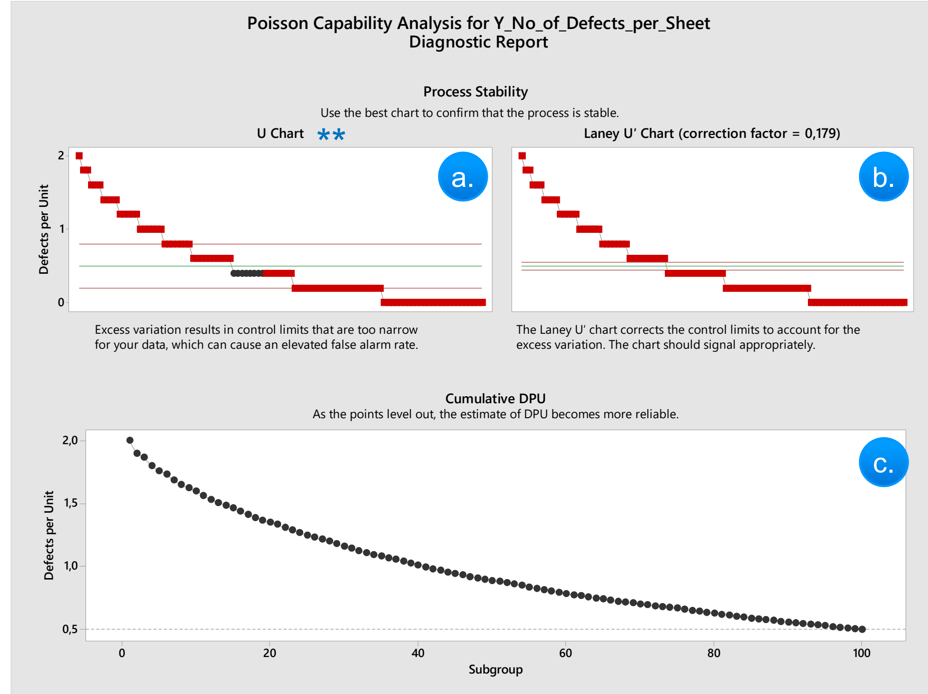
Report Chart with statements about:

a. Stability of the Process:
- Outliers and Patterns in the data indicate low Stability of the Process
> interpret Parameters of Process Capability with reservation

b. Number of Subgroups:
- too few observations (subgroups) indicate short term observation
> interpret Parameters of Process Capability with reservation

c. Expected Variation: Overdispersion and Underdispersion
- Variation in Subgroup Sizes can lead to Overdispersion and false alarms
- Intercorrelation of Subgroup results can lead to Underdispersion and missed signals

d. Amount of data:
- too few data/ small sample size might prevent Significance
> Collect more data



Control Charts to asses the stability of the Process

a. u-Chart for the Process
= Time Series Plot with Upper/ Lower Control Limits (UCL/ LCL)
= Outliers and Patterns indicate deviation from Stability

b. Laney u-Chart for the Process
= corrected u-Chart due to overdispersion
= Outliers and Patterns indicate deviation from Stability

c. Chart with cumulative Defects per Unit (DPU):
- an asymptotic progress of the line indicates the reliability of the estimation of the DPU Process Capability Parameter

Check fulfillment of prerequisite conditions

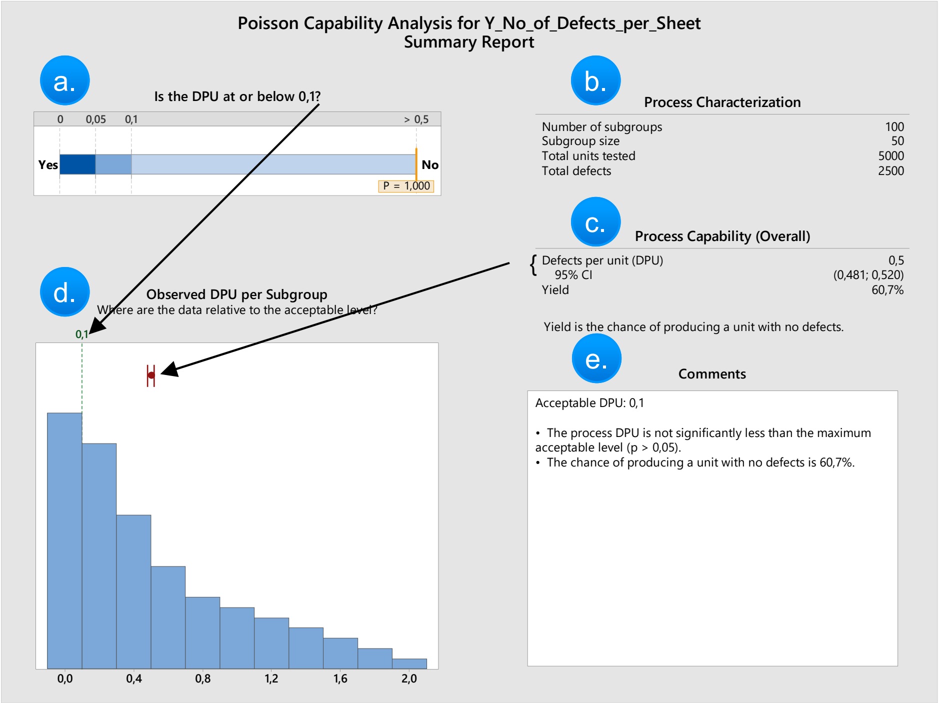
** See also Control Charts: u-Chart

2.

Poisson Process Capability Analysis (3/3)

Results

a.	Significance Test (here: 1p-Test, One-Sample-p-Test): H0: % Defective > USL (Example: 0,1) HA: % Defective <= USL (Example: 0,1) The bar of the chart indicates: - orange line: actual p-value of the Significance Test - dark blue sector: 0% > alpha <= 5% - light blue sector: 5% > alpha <= 10% Example: - p= 1,000 - Confirmation of H0 - DPU is not <= 3% (USL)
b.	Process Characterization: Description of the Sample: - Number of subgroups (Nos) - Subgroup size (SS) - Total items tested (= Nos * SS) Result: - Number of defectives
c.	Process Capability: - Defects per Unit (DPU): 0,5 (= 2500/ 5000) - 95% CI (for DPU) (0,481 - 0,520) - Yield= 60,7%



d.	Observed DPU per Subgroup - Bar Chart with the distribution of DPU/ Subgroup - green dashed line: USL (0,1) - red dot: DPU - red interval: 95% Confidence Interval (CI) for DPU Interpretation: - USL is below 95% CI of % DPU -> indicates confirmation of H0
e.	Comment: - Summary of results and additional hints

Results: DPU, Yield

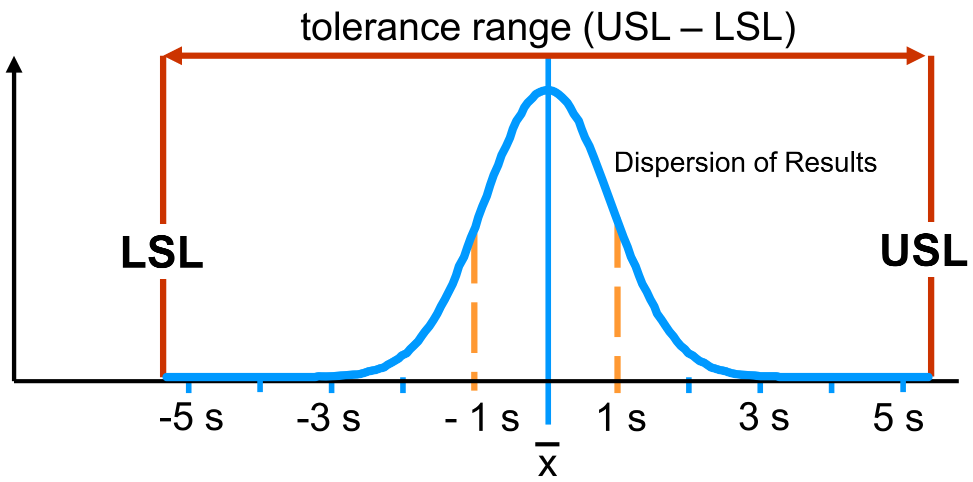
3.

The Cp/ Pp Index relates the Dispersion and …

- **Cp** **= Capability Index**

$$= \frac{USL - LSL}{6s_i}$$

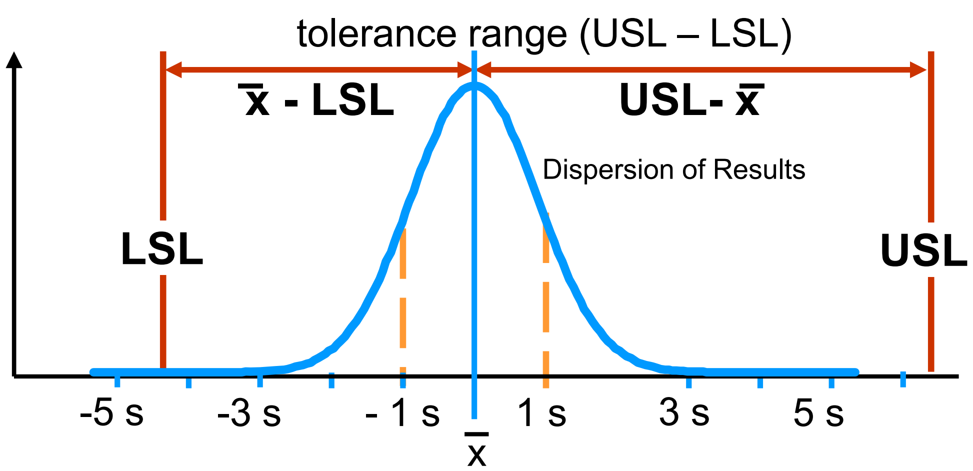
Dispersion-Index Cp shows the tolerance range in relation to the dispersion of the results (Standard Deviation), without considering the center (Mean) of the results.



- **Cpk** **= Capability „katayori" Index** **(= Japanese „center")**

$$= \min \left(\frac{USL - \bar{x}}{3s_i} ; \frac{\bar{x} - LSL}{3s_i} \right)$$

Centering-Index Cpk relates the distance of the center (mean) to the nearest Specification Limit to the dispersion of the results



… Cpk/ Ppk relates the Center of the Results to the nearest Specification Limit

3.

Cpk/ Ppk/ Z Process Capability Analysis (1/4)

Process Capability Analysis

Cpk/ Ppk/ Z Capability

Purpose

Checks whether a certain Process is capable to yield Products/ Services that meet Business/ Customer Requirements.

Description

Product/ Service has a single, cardinal scaled attribute, with an Upper and/ or Lower Specification Limit (as well as a target vale on the scale), which should be met.

Evaluation

Mean and Standard Deviation of the a) Subgroups of the sample (Within) and b) the whole sample (Overall) is related to the Specification Limits/ Target to calculate a) the actual (Cp/ Cpk) and b) the potential (Pp/ Ppk) Capability.

Data

- Values of the attribute of the Product/ Service
- in a series of constant/ variable subgroups of a sample

Specification Limits

- Upper Specification Limit (USL)
- Lower Specification Limit (LSL)
- Target

Result

- Pp/ Ppk (actual Performance)
- Cp/ Cpk (potential Performance)
- Z-Bench/ % out of Spec./ PPM (DPMO)

Probability Mass Function

m= Mean of the Population
s= Standard Deviation of the Population
s2= Variance of the Population

$$f(x \mid \mu, \sigma^2) = \frac{1}{\sqrt{2\sigma^2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

Example

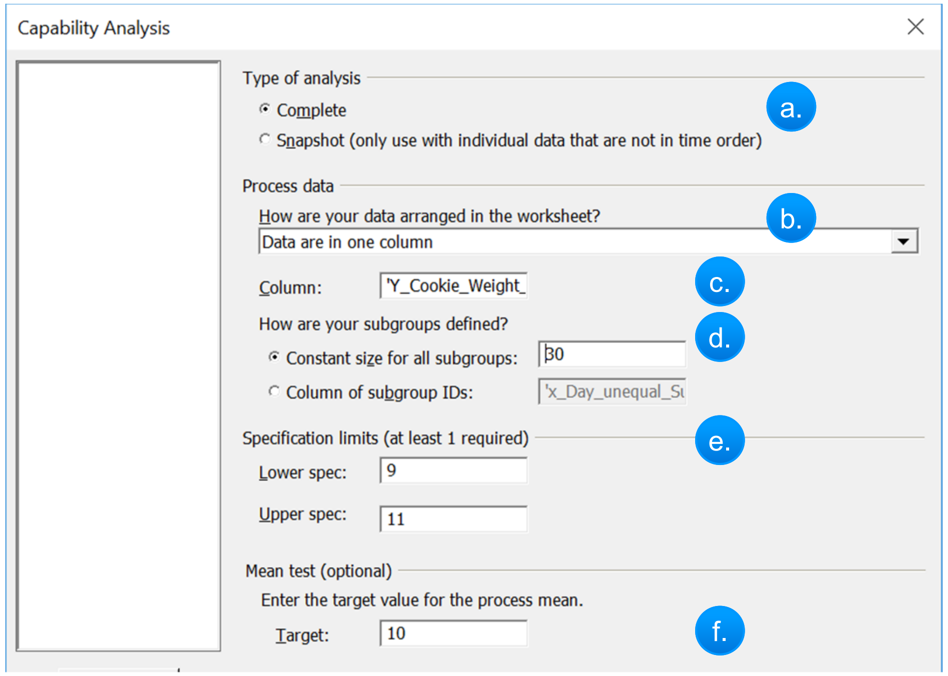
Cycle Time
Viscosity

Weight of Cookies within Specification Limits (9g < Cookie Weight < 11g)

Y	Scale Level
1	cardinal
	Transform Scale Values
	Data of attribute must be normal distributed; if not: a) transform data or b) use alternative Capability Index

Mintab Menu

- Assistant/ Capability Analysis/ Capability Analysis Cpk, Ppk, Z
- Stat/ Quality Tools/ Capability Analysis/ Normal



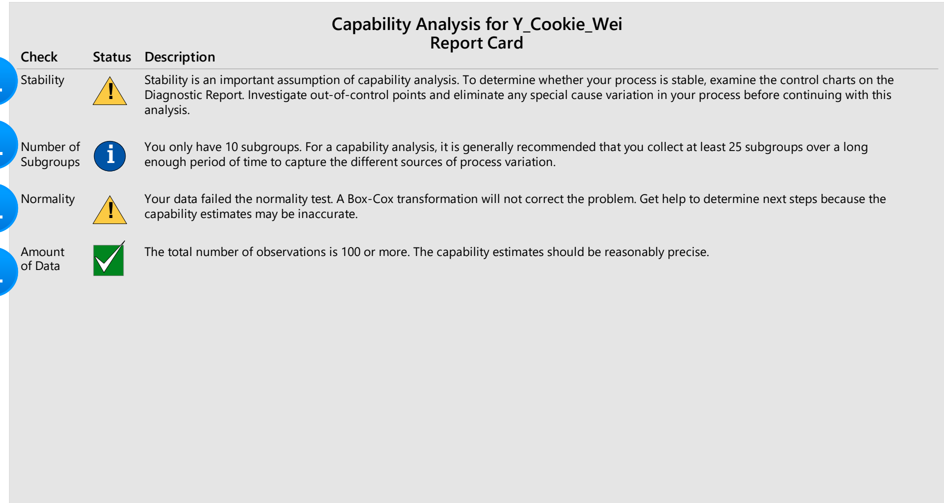
Dialog

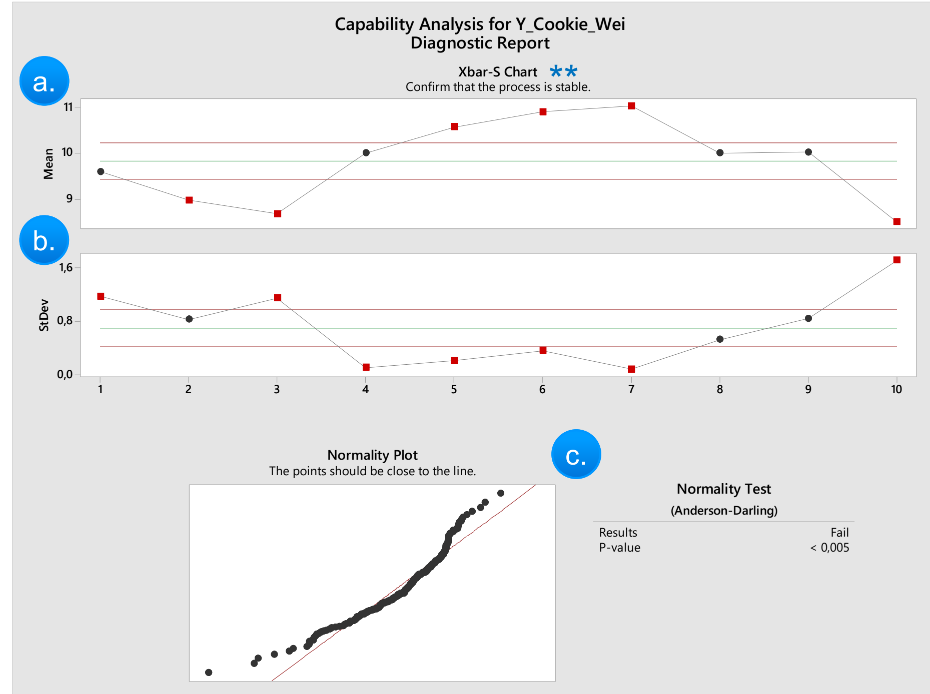
a.	- Complete: if data in sample are collected from a continuous time series - Snapshot: data are arbitrary collected and arranged (data are seldom ND!)
b.	- Data in one column, Subgroups in another column with Subgroup Id´s - Data in multiple columns, with one column for each Subgroup
c.	Y-Variable with single data for each Product/ Service
d.	- Constant Size for all Subgroups: e.g. always 30 Cookies/ sheet - Column with Subgroup ID´s
e.	- Lower Specification Limit (LSL - Upper Specification Limit (USL) in DPU
f.	Target Value (optional; typically between LSL and USL)

Capability for the overall statement: Unit (Opportunity * No. of Product/ Service) is: ok vs. ko

3.

Cpk/ Ppk/ Z Process Capability Analysis (2/4)





Report Chart with statements about:

a. Stability of the Process:
- Outliers and Patterns in the data indicate low Stability of the Process
> interpret Parameters of Process Capability with reservation

b. Number of Subgroups:
- too few observations (subgroups) indicate short term observation
> interpret Parameters of Process Capability with reservation

c. Expected Variation: Overdispersion and Underdispersion
- Variation in Subgroup Sizes can lead to Overdispersion and false alarms
- Intercorrelation of Subgroup results can lead to Underdispersion and missed signals

d. Amount of data:
- too few data/ small sample size might prevent Significance
> Collect more data

Control Charts to asses the stability of the Process

a. Xbar-Chart for the Process
= Time Series Plot with Mean of Subgroups and Upper/ Lower Control Limits (UCL/ LCL)
= Outliers and Patterns indicate deviation from Stability

b. S-Chart for the Process
Time Series Plot with Standard Dedivation (SD) of Subgroups and UCL LCL for SD
= Outliers and Patterns indicate deviation from Stability

c. Normality Chart and Anderson-Darling Normality Test
- Data are not normal distributed (note the bimodal distribution of data)

Check fulfillment of prerequisite conditions

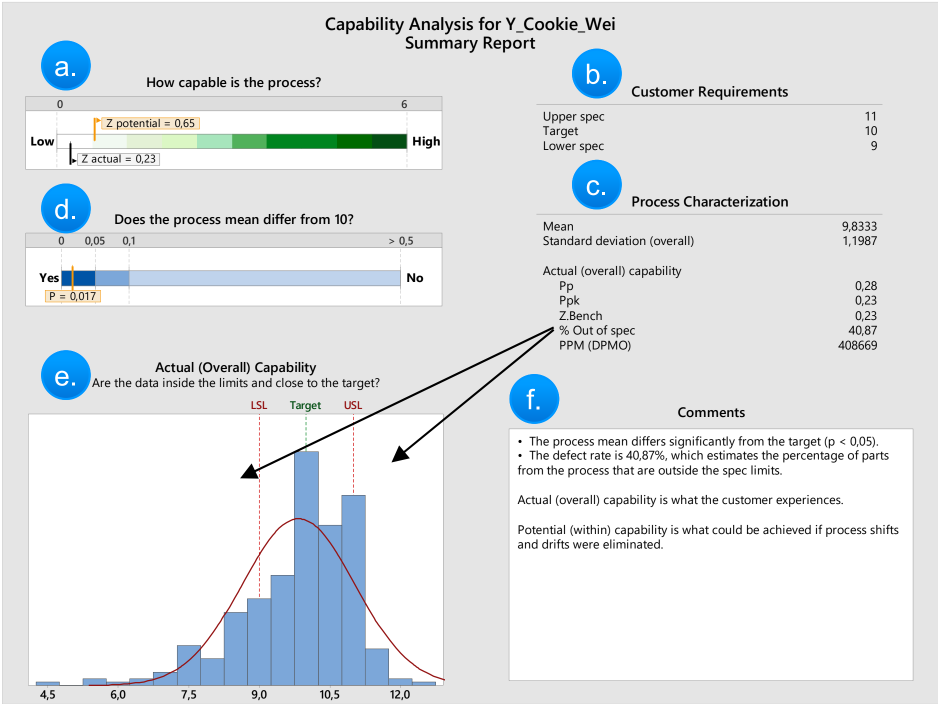
** See also Control Charts: Xbar-S Chart

3.

Cpk/ Ppk/ Z Process Capability Analysis (3/4)

Results

a.	Process Capability: Z actual: = Z-Bench actual = Sigma-Level for the overall Capability based on Ppk, i.e. the overall standard deviation= 0,23 Z potential = Z-Bench potential = Sigma-Level for the within Capability based on Cpk, i.e. the within standard deviation= 0,65 (see next slide)
b.	Customer Requirements: - Upper Specification Limit (USL) - Target - Lower Specification Limit (LSL)
c.	Process Capability: Parameter of the sample: - Mean (overall) - Standard Deviation (overall) Actual (overall) Capability - Pp (dispersion related)= 0,28 - Ppk (centre related)= 0,23 - Z-Bench (actual, i.e. overall)= 0,23 - % Out of Specification= 40,87 (408669/ 1 Mio * 100) - PPM (DPMO)= 408669
d.	Significance Test (here: One-Sample-t-Test): H0: Mean = Target (Example: 10) HA: Mean <> Target (Example: 10) The bar of the chart indicates: - orange line: actual p-value of the Significance Test - dark blue sector: 0% > alpha <= 5% - light blue sector: 5% > alpha <= 10% Example: - p= 0,017 - if alpha= 10%= Rejection of H0, i.e.: Mean differs from Target - if alpha= 5%= Rejection of H0, i.e.: Mean differs from Target



e.	Actual (overall) Capability - Histogram with the distribution of the data - Normal Distribution (Model) based on the Standard Deviation (overall) - LSL, Target, USL - % Out of Specification= 40,87
f.	Comment: - Summary of results and additional hints

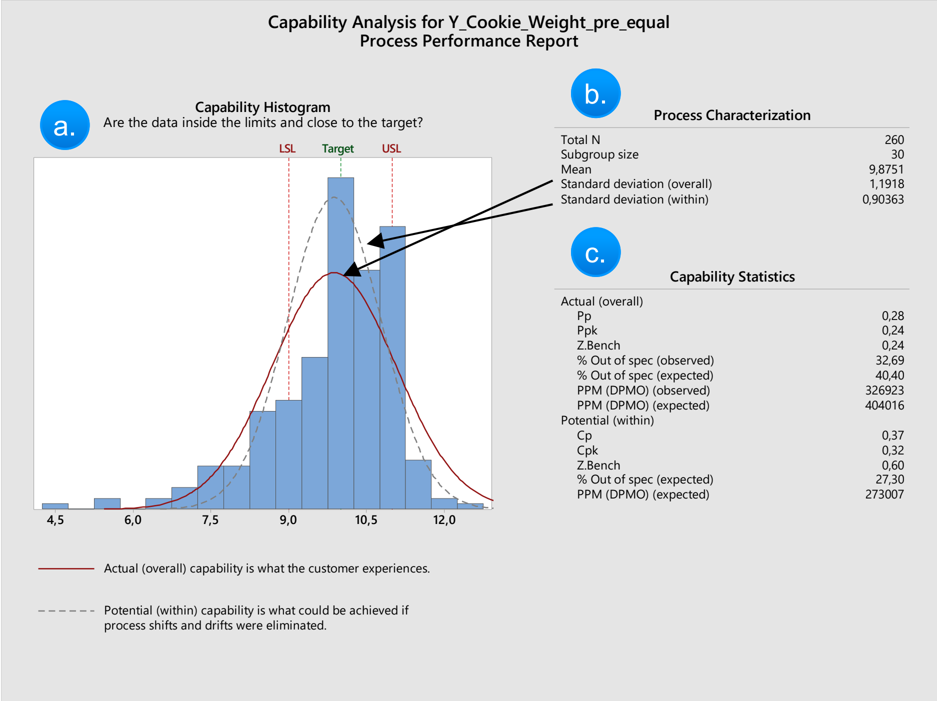
Results: Pp, Ppk, Z-Bench, % Out of Specification, PPM

3.

Cpk/ Ppk/ Z Process Capability Analysis (4/4)

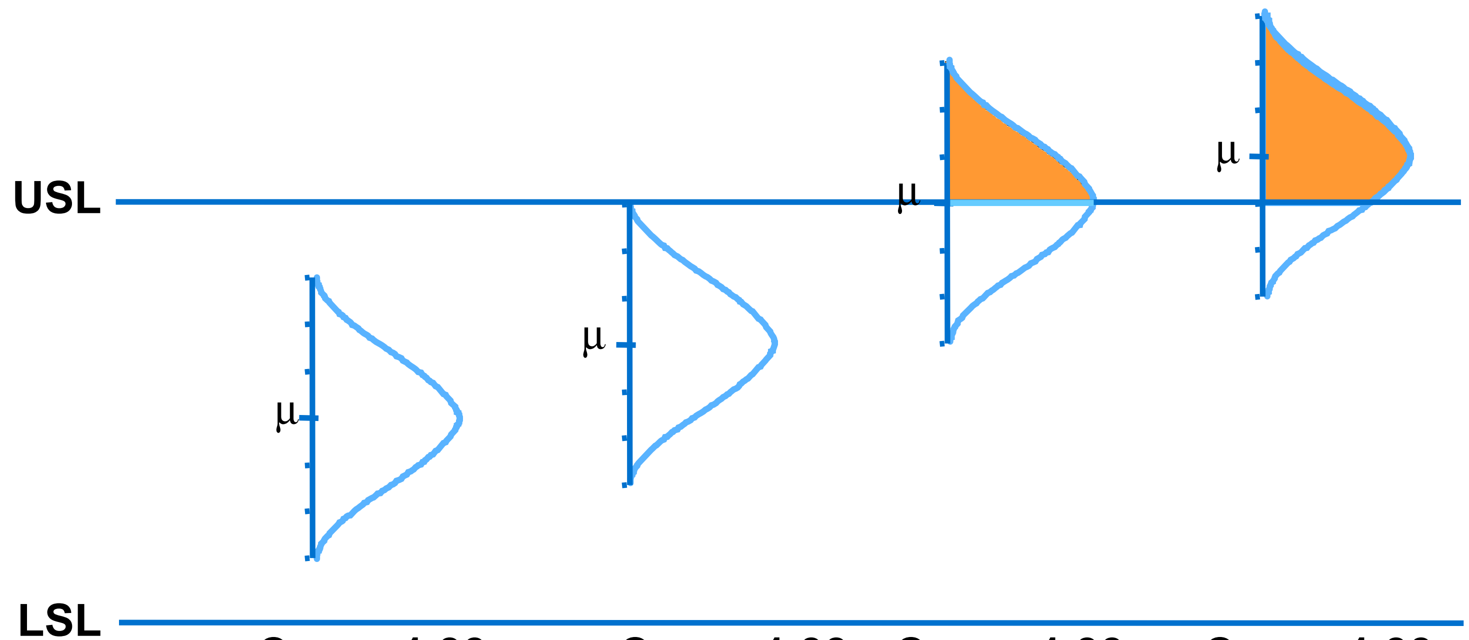
Further Results

a.	Capability Histogram: - distribution of data - LSL, Target, USL - Actual overall Capability, based on the overall Standard Deviation - Actual within Capability, based on the within Standard Deviation (see next slides)
b.	Process Characterization Description of the Sample - Total N= 300 - Subgroup Size= 30 (- Number of Subgroups= 10) Parameter of the Sample: - Mean - Standard Deviation (overall) - Standard Deviation (within)
c.	Capability Statistics: Actual (overall) Capability - Pp (dispersion related)= 0,28 - Ppk (centre related)= 0,23 - Z-Bench)= 0,23 - % Out of Specification (observed)= 34,33 - % Out of Specification (expected)= 40,87 - PPM (DPMO) (observed)= 343333 (in the sample) - PPM (DPMO) (expected)= 408669 (based on the estimation for the Population, with the given overall Standard Deviation) Potential (within) Capability - Cp (dispersion related)= 0,38 - Cpk (centre related)= 0,32 - Z-Bench= 0,65 (see previous slide) - % Out of Specification (expected)= 25,89 - PPM (DPMO) (expected)= 258881(based on the estimation for the Population, with the given overall Standard Deviation)



Further Results: Pp, Ppk, Z-Bench, % Out of Specification, PPM

Relationships between Cp und Cpk:



	Cp = 1,33	Cp = 1,33	Cp = 1,33	Cp = 1,33
	Cpk = 1,33	Cpk = 1,0	Cpk = 0,0	Cpk = -0,33
Defects:	***≈ 63 dpm***	***≈ 1350 dpm***	***50%***	***84%***

If the Process is centered, then Cpk = Cp, in other cases Cpk < Cp

Source: Siemens AG, Kennzahlen zur Prozessqualität

Capability Index and Sigma Level are equivalent

$Cp = 2 = \frac{USL - LSL}{6s}$ ➡ $12s = USL - LSL$ ➡ tolerance range of a 6-Sigma Process = 12s

In a centered Process this is also true for Cpk: (USL – Xbar) = (Xbar – LSL) = tolerance range/ 2 = 6s or cp * 3= Sigma Level

<table>
<tr><th>Cp/ Pp
Cpk/ Ppk</th><th>Meaning and Interpretation</th><th>Sigma-level of the Process</th></tr>
<tr><td>0,33</td><td rowspan="2">Current position still within the defined tolerance range
Current scatter of the values significantly outside of the tolerance

= unacceptable process</td><td>1</td></tr>
<tr><td>0,66</td><td>2</td></tr>
<tr><td>1,00</td><td rowspan="2">Current position within the defined tolerance range
Current scatter of the values significantly within the tolerance range

= acceptable process</td><td>3</td></tr>
<tr><td>1,33</td><td>4</td></tr>
<tr><td>1,67</td><td rowspan="2">Current position centered in the tolerance range
Current scatter of the values much smaller than the defined tolerance

= very good Process</td><td>5</td></tr>
<tr><td>2,00</td><td>6</td></tr>
</table>

The interpretation of the Sigma Level depends on the type of Process

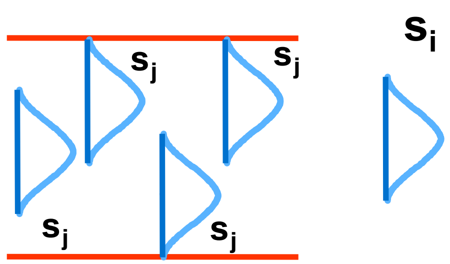
Statements about: Cp / Cpk and Pp / Ppk (1/3)

Prozess Capability Cp, Cpk

on the basis the Standard Deviation (within) s_i

- Short Term Capability of a stable Processes, i.e. the potential of a Process
- Sample based on a several small subgroups or large sample in a time series
- Uses the within Standard Deviation s_i of the sample(s):

$$S_i = \sqrt{\frac{1}{m}\sum_{j=1}^{m} s_j^{2}}$$

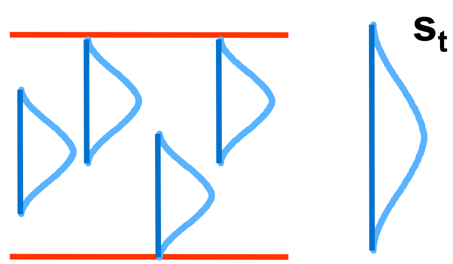


Process Perfomance Pp, Ppk

on the basis the Standard Deviation (overall) s_t

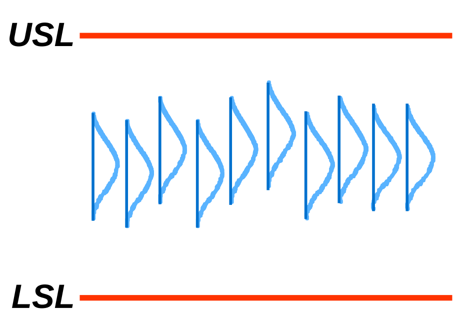
- Long term Capability of a Process
- Sample based on many large subgroups
- Uses the overall Standard Deviation s_t of the data

$$S_t = \sqrt{\frac{1}{m*n-1}\sum_{j=1}^{m}\sum_{i=1}^{n}\left(x_{ij}-\bar{\bar{x}}\right)^{2}}$$

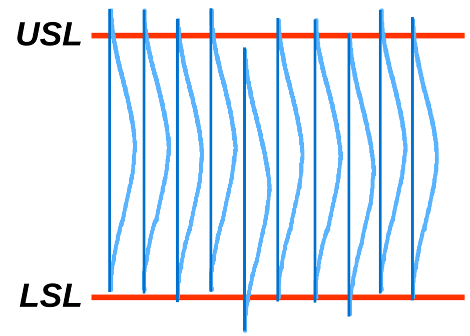


In a stable Process: Pp = Cp and Ppk = Cpk

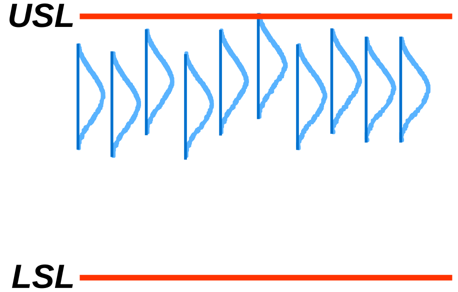
Staements about: Cp / Cpk and Pp / Ppk (2/3)



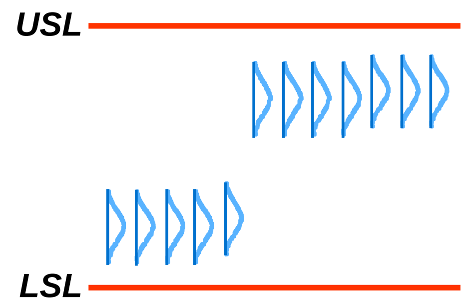
Cp	**good (2)**
Cpk	**good (2)**
Pp	**good (2)**
Ppk	**good (2)**



Cp	**insufficient (6)**
Cpk	**insufficient (6)**
Pp	**insufficient (6)**
Ppk	**insufficient (6)**



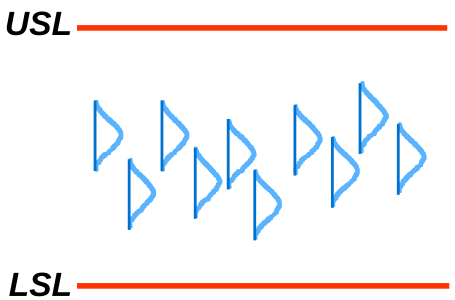
Cp	**good (2)**
Cpk	**sufficient (4)**
Pp	**good (2)**
Ppk	**sufficient (4)**



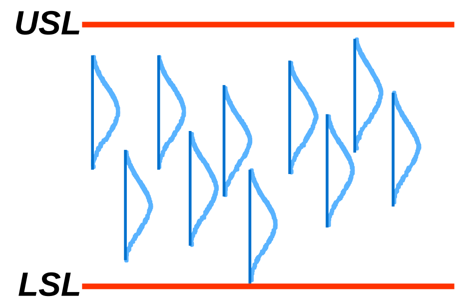
Cp	**very good (1)**
Cpk	**sufficient (4)**
Pp	**deficient (5)**
Ppk	**deficient (5)**

In a stable Process: Pp = Cp and Ppk = Cpk

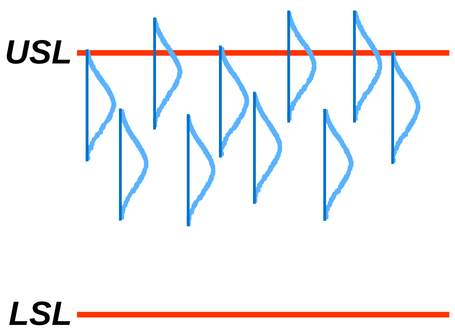
Statements about: Cp / Cpk and Pp / Ppk (3/3)



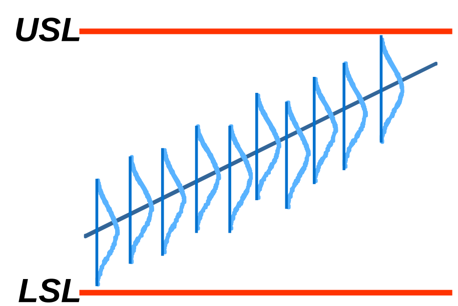
Cp	very good (1)
Cpk	good (2)
Pp	satisfactory (3)
Ppk	satisfactory (3)



Cp	satisfactory (3)
Cpk	sufficient (4)
Pp	deficient (5)
Ppk	deficient 5)



Cp	good (2)
Cpk	deficient (5)
Pp	insufficient (6)
Ppk	insufficient (6)



Cp	good (2)
Cpk	satisfactory (3)
Pp	deficient (5)
Ppk	deficient (6)

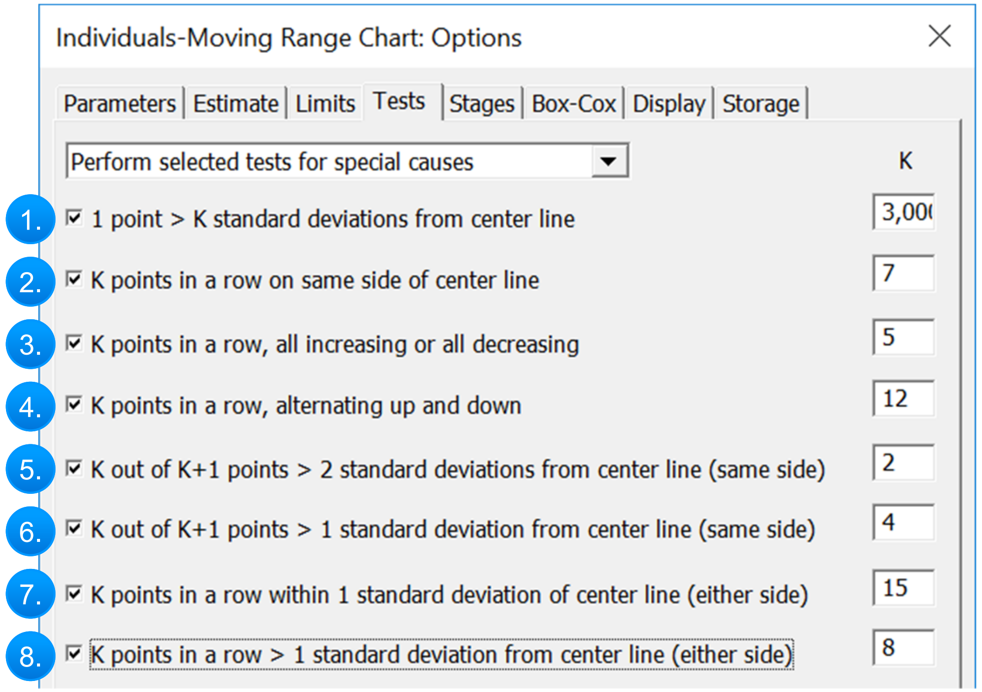
In a stable Process: Pp = Cp and Ppk = Cpk

Quelle: nach Siemens AG, Kennzahlen zur Prozessqualität

Process Control

Details

*The Control Charts offer 8 tests** to identfy Signals …*



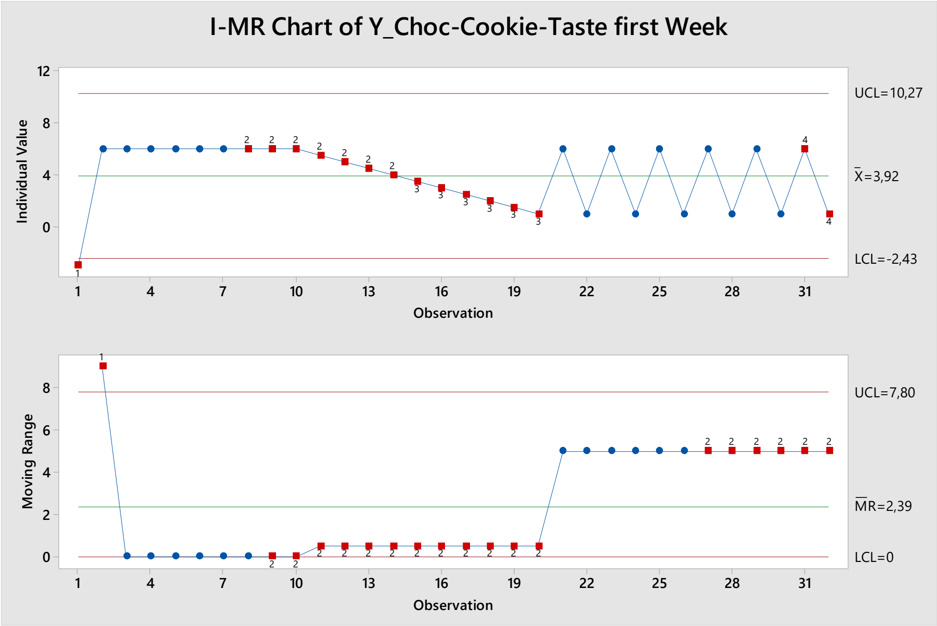
#	Control Chart Tests to Identify Signals	Mintab Assistant tests:
1	1 Point > K Standard deviation** from the center line Identifying the outliers, e.g. rare results Cause: differentiable, single influence on the process	✓
2	K points in a row on the same side of the center line Identifying a series of similar results on the same level of deviation Cause: consistant negative influence (?)	✓
3	K points in a row, all increasing or decreasing Identifying upwards and downward trends Cause: cumulative changeing influence (?)	✓
4	K points in a row, alternating up and down Identifying repeated "Ping-Pongs" Cause: dependency bewtween influences, alternating the direction of the next influence (?)	✓
5	K out of K+1 > 2 standard deviations** from the center line (same side) early notification if something is amiss (:= warning limit; UWL= upper warning limit, LWL= lower warning limit)	✓
6	K out of K+1 > 1 standard deviations** from the center line (same side) early notification if something is amiss (:= warning limit; UWL= upper warning limit, LWL= lower warning limit)	
7	K points in a row within 1 standard deviation** of the center line (either side) early notification of a series of similar results on the same level of deviation Cause: consistant negative influence (?)	
8	K points in a row > 1 standard deviation** from center line (either side) early notification of a series of similar results on the same level of deviation Cause: consistant negative influence (?)	

** applies to the different dispersion parameter of the choosen Chart, like MR, R, …

… like Outliers, Patterns, Trends

** see Statistic/Control Chart/ .../... Options/ Tests

Examples for identified Signals in the I-MR Chart



- Signals in the Ouptuts indicate, that the Process is not under control. Before the Control Limits can reliably be interpreted, the Signals have to be eliminated.
- To do so, every single Signal needs to be inspected by analysing the corresponding Product/ Service for the Root Cause of the Signal.
- The Root Cause needs to be eliminated because a controlled Process shows only random fluctuation and is therefore normal distributed.
- Then apply the Control Chart again and check it for Signals.

Test Results for I Chart of Y_Choc-Cookie-Taste first Week

TEST 1. One point more than 3,00 standard deviations from center line.
Test Failed at points: 1

TEST 2. 7 points in a row on same side of center line.
Test Failed at points: 8; 9; 10; 11; 12; 13; 14

TEST 3. 5 points in a row all increasing or all decreasing.
Test Failed at points: 15; 16; 17; 18; 19; 20

TEST 4. 12 points in a row alternating up and down.
Test Failed at points: 31; 32

Test Results for MR Chart of Y_Choc-Cookie-Taste first Week

TEST 1. One point more than 3,00 standard deviations from center line.
Test Failed at points: 2

TEST 2. 7 points in a row on same side of center line.
Test Failed at points: 9; 10; 11; 12; 13; 14; 15; 16; 17; 18; 19; 20; 27; 28; 29; 30; 31; 32

* WARNING * If graph is updated with new data, the results above may no longer be correct.

Every Signal probably has a specific Root Cause

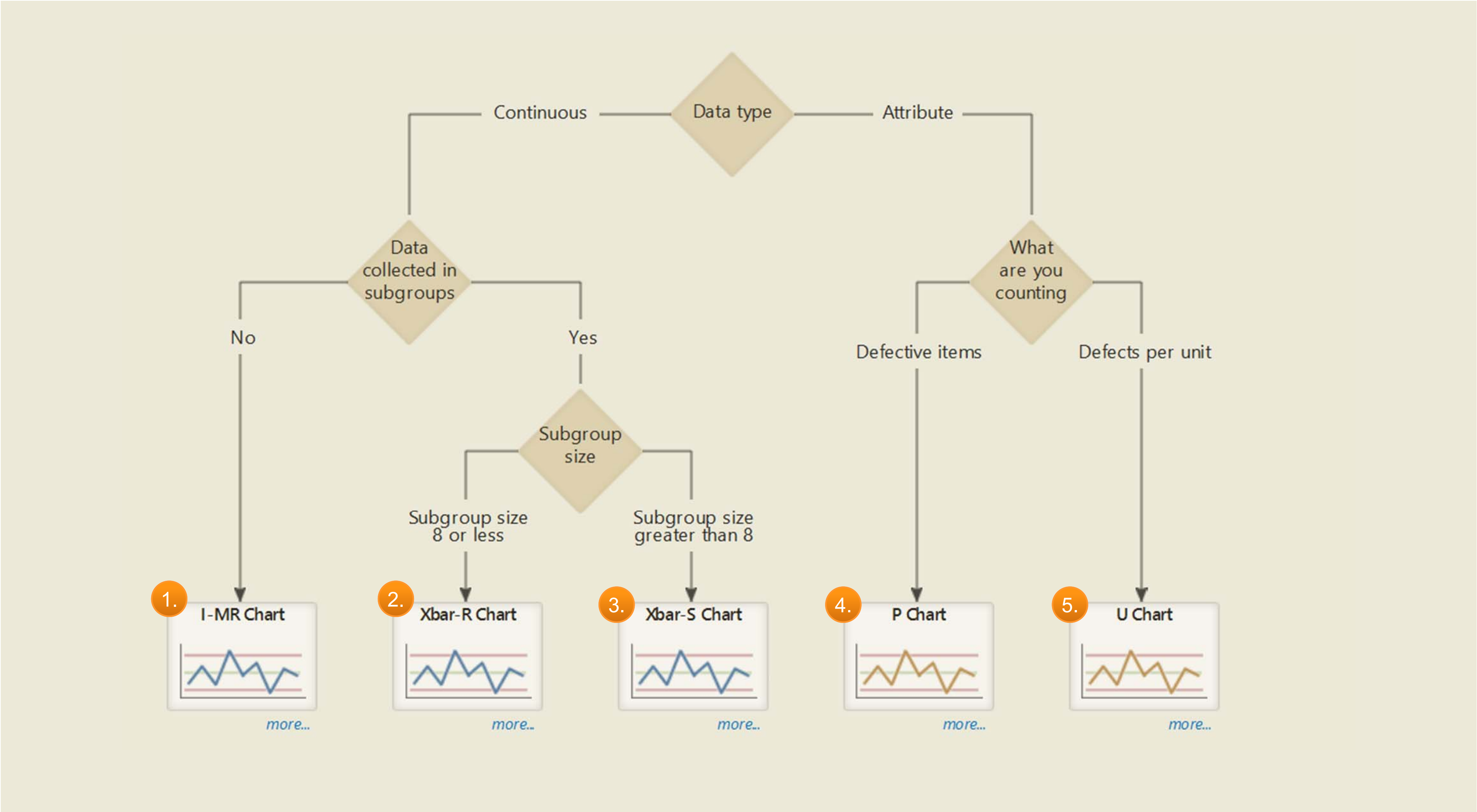
Overview to important Control Charts

#	Control Chart	Purpose: Identify Signals in a sequence of chronological collected ...	Variables and Grouping	Y Scale Level	Mintab Menu Option
1	I/ MR	... Single values in relation to other single values. I-Card: deviation of single values (Individual) from Mean (Xbar) MR-Card: deviation of the difference between two adjacent values (Moving Range), from the average of the difference of all adjacent values (= Mean Moving Range= MRbar)	each value of the variable is represented as one number and one dot in the chart	cardinal	Assistant/ Control Charts/ I-MR Chart
2	Xbar/ R	... Subgroups in relation to other subgroups (N_Subgroup <=8). Xbar Chart: deviation of the Mean of the subgroup (Xbar_Subgroup) from the overall Mean (Xbar_Sample) R-Card: Deviation of the Range per subgroup (R) from overalll average Range (Rbar)	each value of the variable is pooled with adjacent values to a subgroup, with a dot for each subgroup - Number of values per subgroup: <= 8; - Size of the subgroup: constant or variable	cardinal	Assistant/ Control Charts/ Xbar-R Chart
3	Xbar/ S	... Subgroups in relation to other subgroups (N_Subgroup >8). Xbar Chart: deviation of the Mean of the subgroup (Xbar_Subgroup) from the overall Mean (Xbar_Sample) R-Chart: Deviation of the Standard Deviations per subgroup (S) from the overall Standard Deviation (Squer)	each value of the variable is pooled with adjacent values to a subgroup, with a dot for each subgroup - Number of values per subgroup: > 8; - Size of the subgroup: constant or variable	cardinal	Assistant/ Control Charts/ Xbar-S Chart
4	p-Chart	... Amount of errors in one subgroup compared to the Amount of errors in other subgroups. pbar-Chart: deviation of the percentage of defective Units of every subgroup (p_Subgroup) from the average deviation of defective Units of all subgroups (pbar)	each value of the variable represents the number of defective Units in a subgroup, with a dot for each subgroup - Number of values per subgroup: > 5; - Size of the subgroup: constant or variable	nominal	Assistant/ Control Charts/ P Chart
5	u-Chart	... amount of Defects per Unit in relation to the average Defects per Unit. ubar-Chart: deviation of the Defects per Unit (DPU) from the average amount of Defects per Unit (ubar, specifically: DPUbar)	each value of the variable represents the number of Defects per Unit, with a dot for each Opprtunities subgroup - Number of Opprtunities: > 5 - Number of Opportunities can be: constant or variable	nominal	Assistant/ Control Charts/ U Chart

** for the type of identified Signals (:= systematic results) refer to slide:
Examples for identified Signals in the I-MR Chart

...

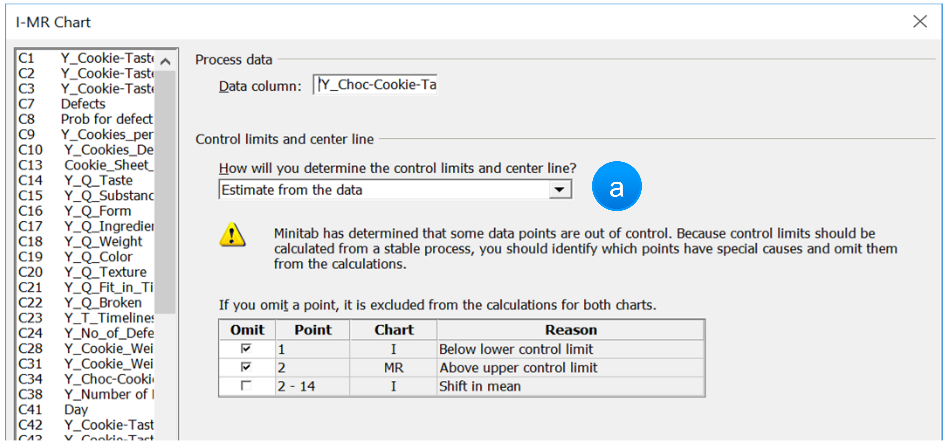
Selection of the suitable Control Chart with the Minitab Assistant

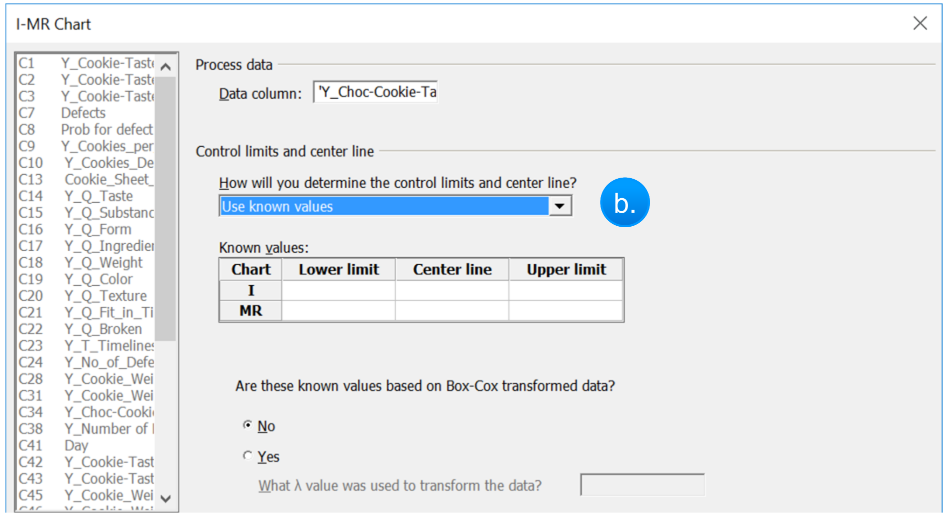


...

Two options for the Calculation of the Control Limits and the Center Line in the Control Charts

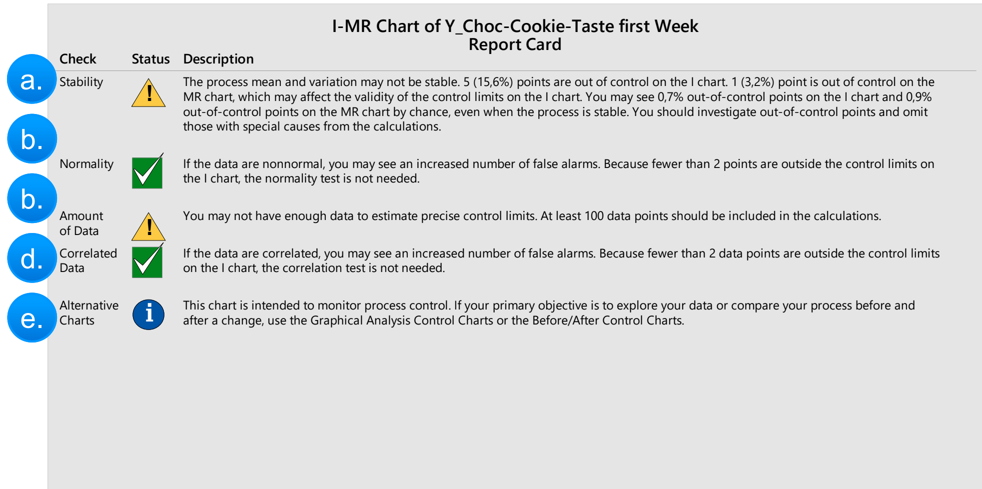
a. Estimate from the data	Recommended for first analyses of a Process. Mintab then detects systematic deviations and indicates them in a table before displaying the Chart. Exclude these suspicious values (Signals) to only include the presumably controlled values in the calculation of the Control Limits and thus to probably indicate them as Signals in the Charts. Not excluding these values means to include these Signals into the calculation of the Control Limits. This might broaden the limits and e.g. Outliers might not be detected, because they are handled as normal/ controlled values and might fall within the Control Limits after calculation. Take care, that not more than 20% of the values are excluded to assure a reliable calculation (recommended).
b. Use known values	Recommended for Processes, which are or have been under control. - Enter the historic values of a controlled Process and enter the already calculated values for the Control Limit and Center Line. - If a Process does not meet the Customer Requirements, then calculate an additional Control Chart and enter the Target value as Center Line and the Specification Limits as Control Limits, to show the Process Performance in relation to the Customer Requirements.
	If the data have been Box-Cox transformed, to normalize non-normal distributed values then the resulting Lambda-Value should be entered here.

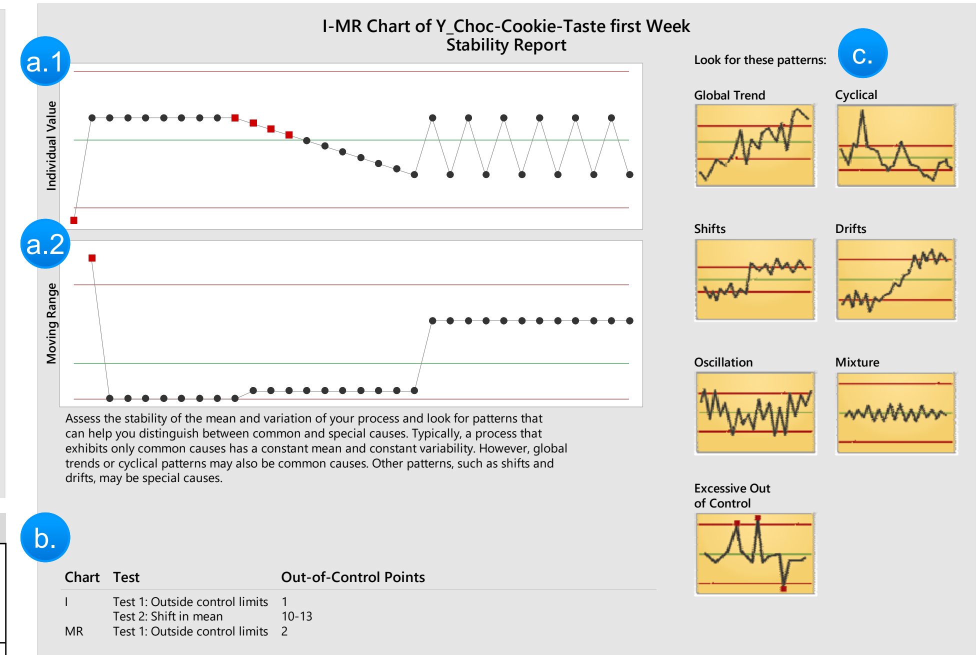




a. Estimate Control Limits and Center Line or b. use known, i.e. historical data

The Report Card and the Stability Report are very similar for all Control Charts





Report Chart with statements about:

a.	Stability of the Process: - Outliers and Patterns in the data indicate low Stability of the Process > interpret Parameters of Process Capability with reservation
b.	Normality: - If the data are not normal distributed, then a > Box-Cox Transformation might help to align the data, especially for the Xbar-S Chart. Unfortunately, also after this transformation the data are still not normally distributed.
c.	Amount of Data: - too few observations might lead to too narrow corridor between the Control Limits and a wrong position of the Center Line > a Sample Size of at least 100 values is recommended
d.	Correlated Data: - correlated values within a time series might lead, especially for extreme values, to false alarms, i.e. a sequence of Outliers which depend on the cause of the correlation > identify the cause of the correlation, i.e. the dependency between values
e.	Alternative Charts: - hints about alternatives to the selected Control Chart

Stability Report

a.	The Charts show the: a.1: Deviation of the data from the Center (Line of the overall Mean) a.2: Variability between adjacent values (I/MR) or Dispersion within Subgroups (Xbar-R, Xbar-S) Signal are indicated by:
b.	Explanation of indicated Signals by the underlying test (see also slide: Control Charts > Tests)
c.	Further patterns (Signals), beside the tested and identified Signals > Identify Root Causes of this Patterns, if they occur

Check these information and follow the advices

1.

I/MR-Chart: monitor cardinal scaled single values of a time series

Control Chart

I/ MR

Purpose:
Identify Signals in a sequence of chronological collected ...
... Single values in relation to other single values.

I-Card: deviation of single values (Individual) from Mean (Xbar)
MR-Card: deviation of the difference between two adjacent values (Moving Range), from the average of the difference of all adjacent values (= Mean Moving Range= MRbar)

Variables and Grouping

Y	Single value
Data Type	cardinal
Relation	each value of the variable is represented as one number and one dot in the chart

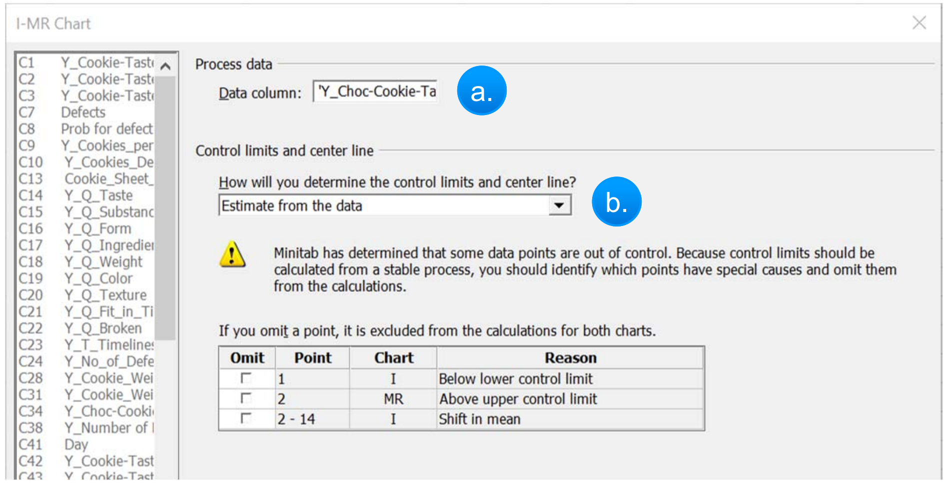
Example

Monitoring the accessibility by phone/ day
Monitoring fuel consumption/ 100km

Monitoring the amount of Cookies/ baking sheet (backing time/ backing sheet)

Mintab Menu Commands

Assistant/ Control Charts/ I-MR Chart

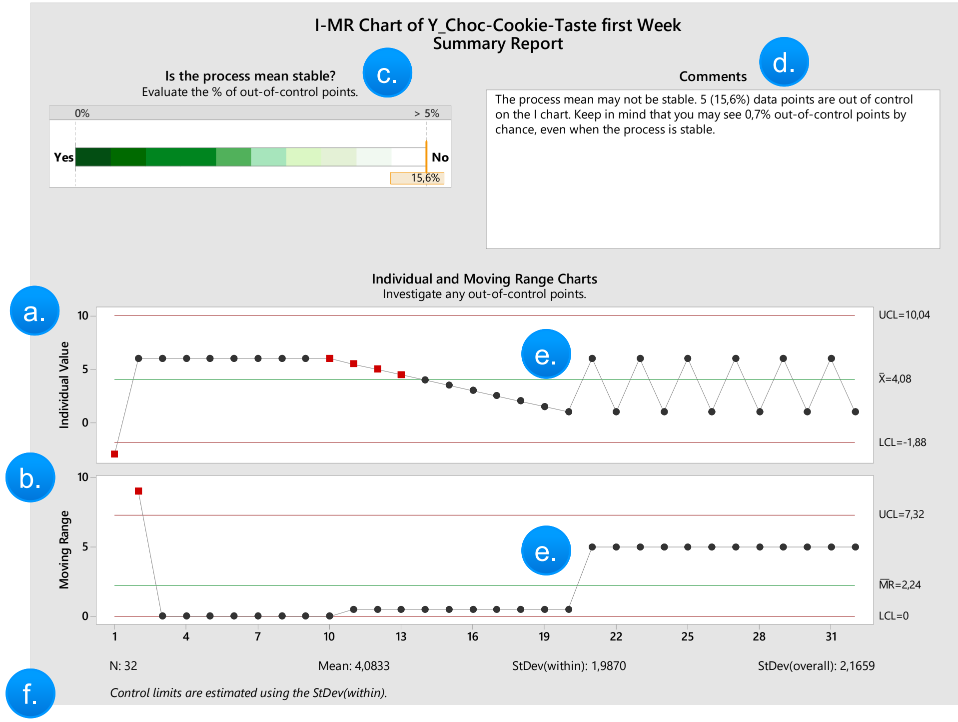


Dialog	
a.	Data Column: Y- Variable with single values, observed in a consecutive time series Recommended for usage if N< 50, otherwise charts are blacked with dots
b.	Determination of Control Limits and Center Line: see slide: Two options for the Calculation of the Control Limits and the Center Line

Example: monitor the Taste of single Cookies

1.

I/MR-Chart: monitor cardinal scaled single values of a time series



Results: Charts	
a. I-Chart	Parameter for the Center of the individual values and the corresponding Control Limits
Xbar	$\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n}$
f. UCL/ LCL	$UCL = \bar{x} + 2{,}66 * \overline{MR}$ $LCL = \bar{x} - 2{,}66 * \overline{MR}$ $\overline{MR} = \frac{\sum_{i=2}^{n} \lvert x_i - x_{i-1} \rvert}{n-1}$
b. MR-Chart	Parameter for the Dispersion of the individual values and the corresponding Control Limits
MR	$\overline{MR} = \frac{\sum_{i=2}^{n} \lvert x_i - x_{i-1} \rvert}{n-1}$
f. UCL/ LCL	$UCL = 3{,}267 * \overline{MR}$ $LCL = 0$

Result: Comments (2/2)	
e.	Dots in Chart Each dot in both Charts represents a single value of the data. If the Charts are blacked out due to a too large number of dots, then choose the Xbar-R (Grouping with <= 8 values) or the Xbar-S Chart (Grouping with > 8 values)
f.	note: Difference between Procedures in the Assistant and the Stats menu: - the Minitab Assistant uses the Standard Deviation (within) to determine the Control Limits

Result: Comments (1/2)	
c.	Process stability statement: - displays the percentage of values out of control - the coloured bar indicates, whether the Process is under (yes) or out of control (no)
d.	Comments: Summary and comments about results

Example: monitor the Taste of single Cookies

2.

Xbar-R Chart: monitor cardinal scaled and grouped values (n_Subgroup <=8)

Control Chart

Xbar/ R

Purpose:
Identify Signals in a sequence of chronological collected ...

... Subgroups in relation to other subgroups (N_Subgroup <=8).

Xbar Chart: deviation of the Mean of the subgroup (Xbar_Subgroup) from the overall Mean (Xbar_Sample)
R-Card: Deviation of the Range per subgroup (R) from overalll average Range (Rbar)

Variables and Grouping

Y	values summarized in subgroups (Size <=8)
Data Type	cardinal
Relation	each value of the variable is pooled with adjacent values to a subgroup, with a dot for each subgroup - Number of values per subgroup: <= 8; - Size of the subgroup: constant or variable

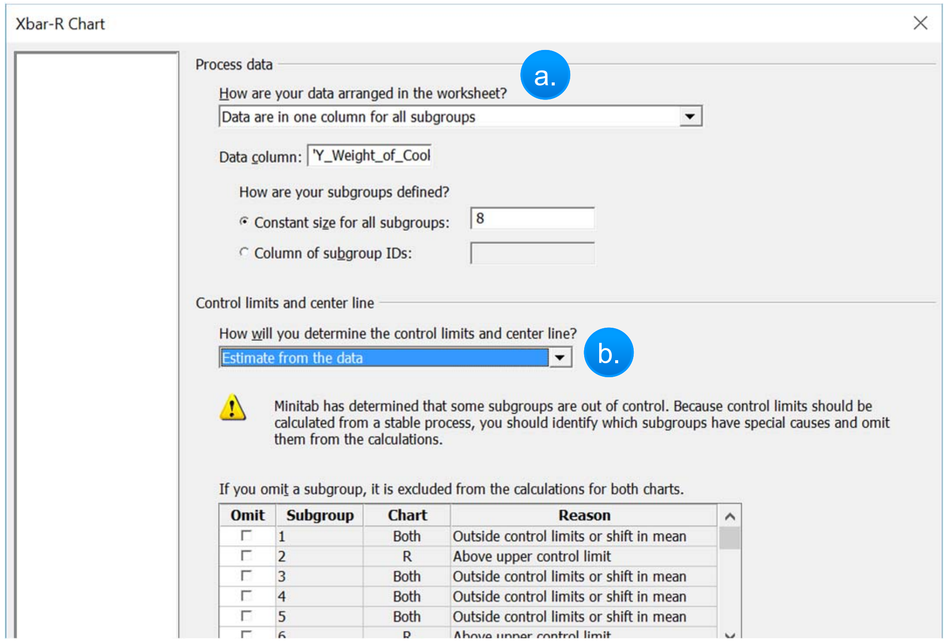
Example

Monitoring the accessibility by phone/ day (Subgroup: week)
Monitoring the jogging time for a specific route (Subgroup: week)

Monitoring the weight of the Cookies (Subgroup: tin with 8 Cookies)

Mintab Menu Commands

Assistant/ Control Charts/ Xbar-R Chart

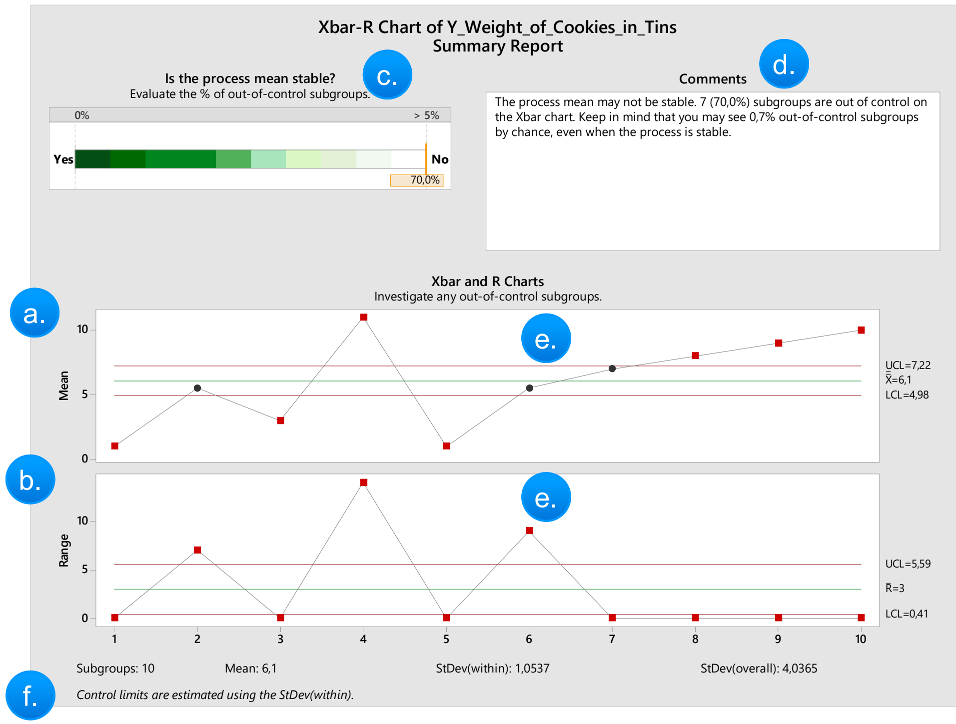


Dialog	
a.	Data Column: - Y-Variable with single values, observed in a consecutive time series - Data will be pooled in Subgroups, size for Subgroups can be <= 8, e.g. one value for each day of the week Definition of Subgroup: see slide: Grouped data can be arranged in two alternative ways
b.	Determination of Control Limits and Center Line: see slide: Two options for the Calculation of the Control Limits and the Center Line

Example: monitor the weight of eight Cookies boxed in a Tin (Subgroup)

2.

Xbar-R Chart: monitor cardinal scaled and grouped values (n_Subgroup <=8)



Results: Charts	
a. Xbar-Chart	Parameter for the Center of the grouped values and the corresponding Control Limits
Xbar	$\bar{\bar{x}} = \frac{\sum_{i=1}^{m} \bar{x}}{m}$ | $\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n}$
f. UCL/ LCL	$UCL = \bar{\bar{x}} + A_2 * \bar{R}$ $LCL = \bar{\bar{x}} - A_2 * \bar{R}$ | $R = x_{max} - x_{min}$ $\bar{R} = \frac{\sum_{i=1}^{n}(R_{max} - R_{min})}{n}$ (A2 is more stable and is based on the normal distribution)
b. R-Chart	Parameter for the Dispersion of the grouped values and the corresponding Control Limits
Rbar	$\bar{R} = \frac{\sum_{i=1}^{n}(R_{max} - R_{min})}{n}$
f. UCL/ LCL	$UCL = D_4 * \bar{R}$ $LCL = D_3 * \bar{R}$ (D3/ D4 are two variables, based on the Normal Distribution to correct the Control Limits)

Result: Comments (1/2)	
c.	Process stability statement: - displays the percentage of values out of control - the coloured bar indicates, whether the Process is under (yes) or out of control (no)
d.	Comments: Summary and comments about results

Result: Comments (2/2)	
e.	The dots in both Charts represent the subgroups with values of the data. If the Charts are blacked out with a too large number of dots, then the Xbar-S Chart (Grouping with > 8 dots)
f.	note: Difference between Procedures in the Assistant and the Stats menu: - the Minitab Assistant uses the Standard Deviation (within) to determine the Control Limits

Example: monitor the weight of eight Cookies boxed in a Tin (Subgroup)

3.

Xbar-S Chart: monitor cardinal scaled and grouped values (n_Subgroup >8)

Control Chart

Xbar/ S

Purpose:
Identify Signals in a sequence of chronological collected ...
... Subgroups in relation to other subgroups (N_Subgroup >8).

Xbar Chart: deviation of the Mean of the subgroup (Xbar_Subgroup) from the overall Mean (Xbar_Sample)
R-Chart: Deviation of the Standard Deviations per subgroup (S) from the overall Standard Deviation (Squer)

Variables and Grouping

Y	values summarized in subgroups (Size >8)
Data Type	cardinal
Relation	each value of the variable is pooled with adjacent values to a subgroup, with a dot for each subgroup - Number of values per subgroup: > 8; - Size of the subgroup: constant or variable

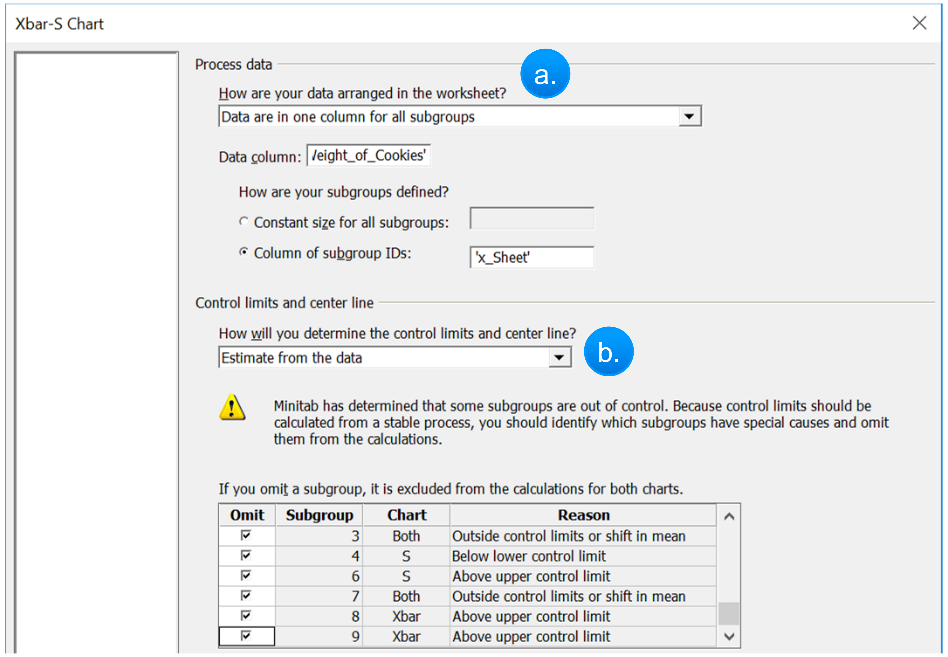
Example

Monitoring inbound calls (Subgroup: 30 min.)
Monitoring of daily expenses (Subgroup: Month)

Monitoring the weight of the Cookies of Cookies per Sheet (Subgroup: Sheet with about 30 Cookies)

Mintab Menu Commands

Assistant/ Control Charts/ Xbar-S Chart

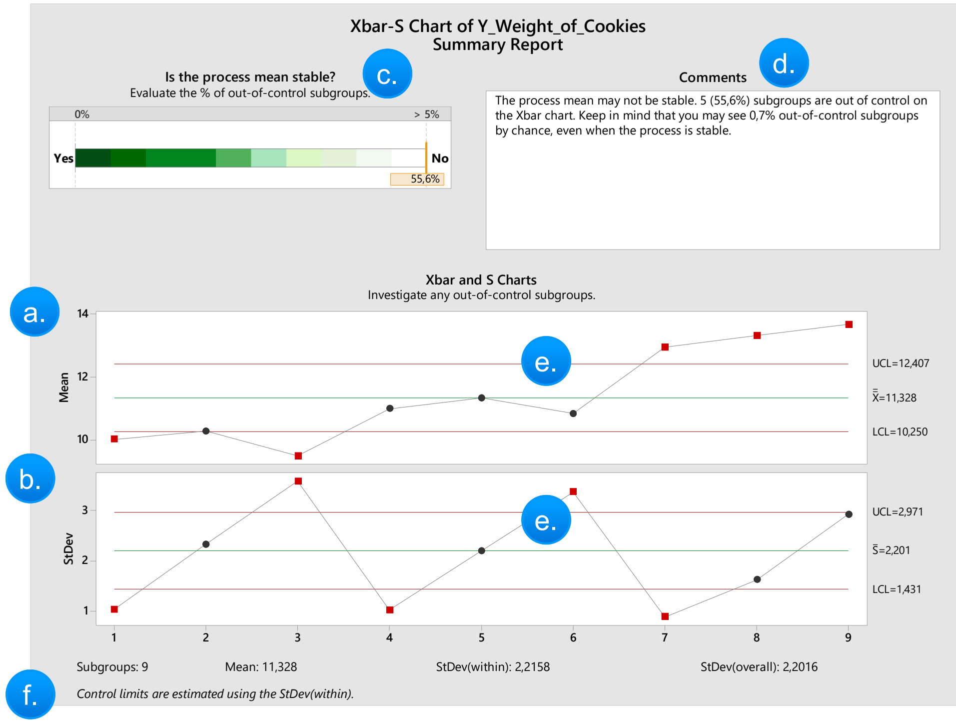


Dialog	
a.	Data Column: - Y-Variable with single values, observed in a consecutive time series - Data will be pooled in Subgroups, size for Subgroups can be > 8, e.g. all Cookies of a day pooled to a subgroup each Definition of Subgroup: see slide: Grouped data can be arranged in two alternative ways
b.	Determination of Control Limits and Center Line: see slide: Two options for the Calculation of the Control Limits and the Center Line

Example: monitor the weight of ca. 30 Cookies on a sheet (Subgroup)

3.

Xbar-S Chart: monitor cardinal scaled and grouped values (n_Subgroup >8)



Results: Charts		
a. Xbar-Chart	Parameter for the Center of the grouped values and the corresponding Control Limits	
Xbar	$\bar{\bar{x}} = \frac{\sum_{i=1}^{m} \bar{x}}{m}$	$\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n}$
f. UCL/ LCL	$UCL = \bar{\bar{x}} + 3 * s$ $LCL = \bar{\bar{x}} - 3 * s$	$s = \sqrt{\frac{\sum_{i=1}^{n}(x_i - \bar{x})}{n-1}}$
b. S-Chart	Parameter for the Dispersion of the grouped values and the corresponding Control Limits	
S-bar	$\bar{s} = \frac{\sum_{i=1}^{m} s_i}{m}$	$s = \sqrt{\frac{\sum_{i=1}^{n}(x_i - \bar{x})}{n-1}}$
f. UCL/ LCL	UCL= $B_4 * \bar{S}$ $LCL = B_3 * \bar{S}$ (B3/ B4 are two variables, based on the Normal Distribution to correct the Control Limits)	

Result: Comments (1/2)	
c.	Process stability statement: - displays the percentage of values out of control - the coloured bar indicates, whether the Process is under (yes) or out of control (no)
d.	Comments: Summary and comments about results

Result: Comments (2/2)	
e.	The dots in both Charts represent the subgroups with values of the data. If the Charts are blacked out with a too large number of dots, then combine more values to a group or narrow the time interval
f.	note: Both Procedures in the Assistant and the Stats menu use the Standard Deviation (within) to determine the Control Limits

Example: monitor the weight of ca. 30 Cookies on a sheet (Subgroup)

4.

P Chart: monitor nominal scaled and grouped defective Units (n_Subgroup >5)

Control Chart

p-Chart

Purpose:
Identify Signals in a sequence of chronological collected ...

... Amount of errors in one subgroup compared to the Amount of errors in other subgroups.

pbar-Chart: deviation of the percentage of defective Units of every subgroup (p_Subgroup) from the average deviation of defective Units of all subgroups (pbar)

Variables and Grouping

Y	Amount of defective units
Data Type	nominal
Relation	each value of the variable represents the number of defective Units in a subgroup, with a dot for each subgroup - Number of values per subgroup: > 5; - Size of the subgroup: constant or variable

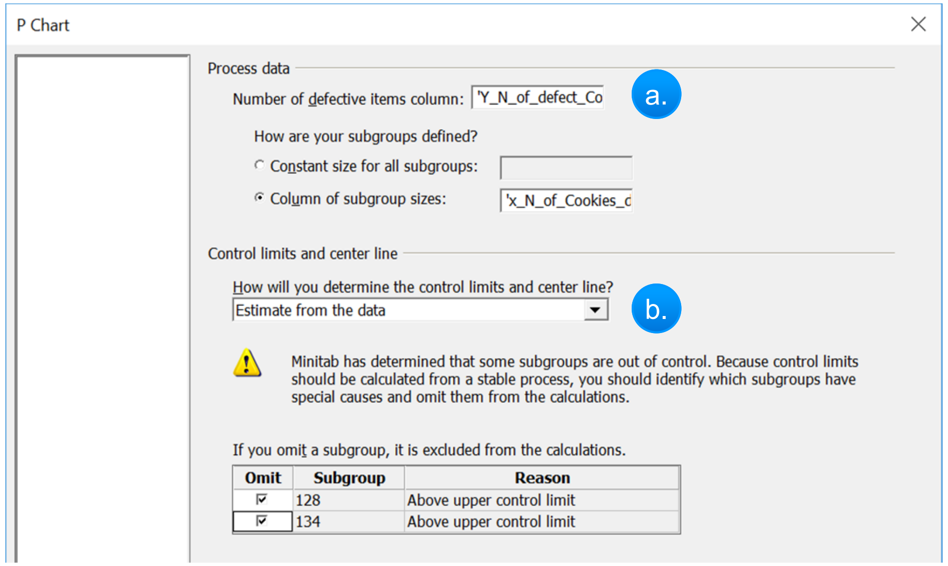
Example

Monitoring of wrong transferred calls per day
Monitoring of the portion of privately spent time in the Internet per day

Monitoring the defective Cookies/ baking sheet

Mintab Menu Commands

Assistant/ Control Charts/ P Chart

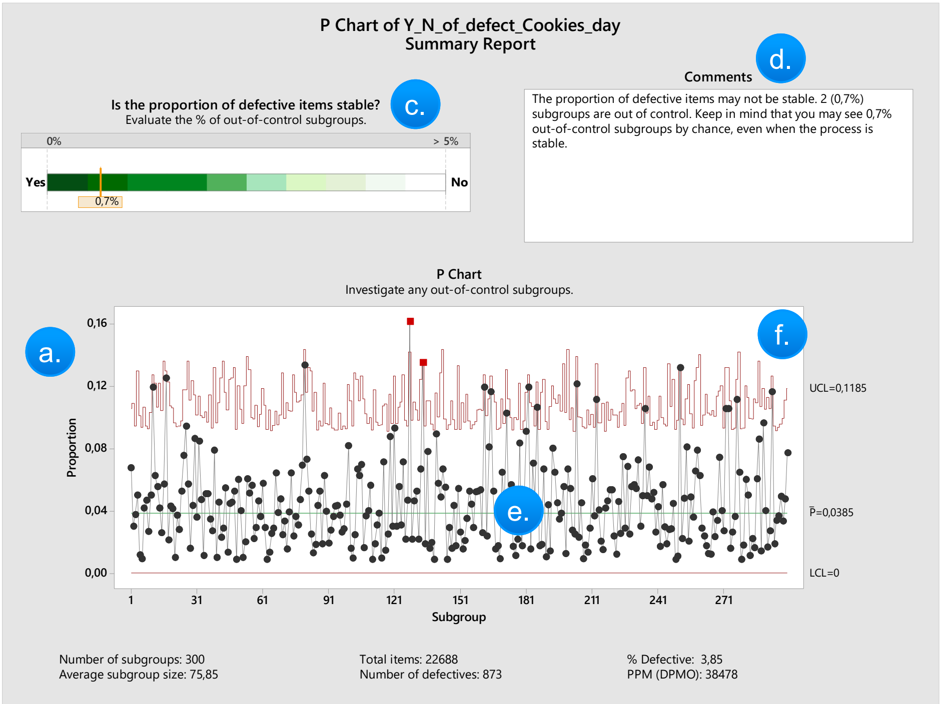


Dialog	
a.	Data Column: - Y-Variable with single values of the number of defective Products/ Services, observed in a consecutive time series - Data will be pooled in Subgroups, size for Subgroups should be > 5 Definition of Subgroup: see slide: Grouped data can be arranged in two alternative ways
b.	Determination of Control Limits and Center Line: see slide: Two options for the Calculation of the Control Limits and the Center Line

Example: monitor the defective Cookies per day (Subgroup)

4.

P Chart: monitor nominal scaled and grouped defective Units (n_Subgroup >5)



Results: Charts	
a. p-chart	Parameter for the Center of the grouped values and the corresponding Control Limits
p-bar	$\bar{p} = \frac{\sum_{i=1}^{n} \frac{defect\ Units\ per\ Subgroup_i}{n\_Units\ per\ Subgroup_i}}{n}$
UCL/ LCL	$UCL = \bar{p} + 3\sqrt{\frac{\bar{p} * (1-\bar{p})}{\bar{n}}}$ $LCL = 0$
b. Variation chart	not defined

Result: Comments (1/2)	
c.	Process stability statement: - displays the percentage of values out of control - the coloured bar indicates, whether the Process is under (yes) or out of control (no)
d.	Comments: Summary and comments about results

Result: Comments (2/2)	
e.	The dots in both Charts represent the subgroups with values of the data. If the Charts are blacked out with a too large number of dots, then combine more values to a group or narrow the time interval
f.	note: The variable Upper Control Limit results from the variable Subgroup size, because the Control Limits depend on a) the % Defective and b) the size of each Subgroup

Example: monitor the defective Cookies per day (Subgroup)

5.

U Chart: monitor nominal scaled Defects per Units

Control Chart

u-Chart

Purpose:
Identify Signals in a sequence of chronological collected ...

... amount of Defects per Unit in relation to the average Defects per Unit.

ubar-Chart: deviation of the Defects per Unit (DPU) from the average amount of Defects per Unit (ubar, specifically: DPUbar)

Variables and Grouping

Y	Amount of defects per unit
Data Type	nominal
Relation	each value of the variable represents the number of Defects per Unit, with a dot for each Opprtunities subgroup - Number of Opprtunities: > 5 - Number of Opportunities can be: constant or variable

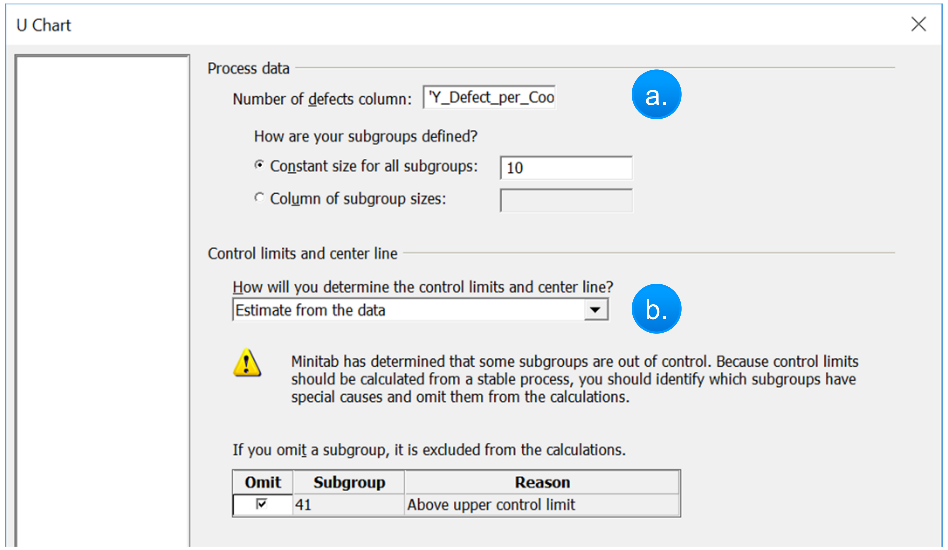
Example

Monitoring of permission changes per call
Monitoring of traffic jams per highway section

Monitoring the amount of different errors per Cookie

Mintab Menu Commands

Assistant/ Control Charts/ U Chart

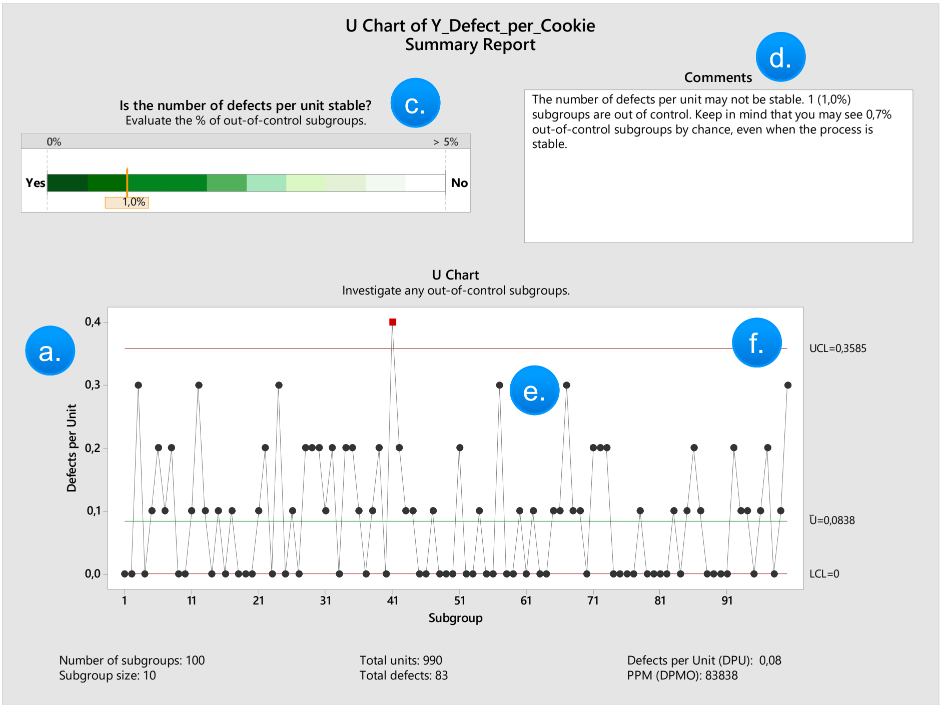


Dialog	
a.	Data Column: - Y-Variable with the number of Defects/ Unit, observed in a consecutive time series, e.g. the number of defects per Cookie Definition of Subgroup: Opportunites for defect, which can be - constant: e.g. always 10 Opportunites for Defect in the same Product/ Service - variable: e.g. different "complex" Cookies measured, with a variable number of Opps.
b.	Determination of Control Limits and Center Line: see slide: Two options for the Calculation of the Control Limits and the Center Line

Example: monitor the Defects per Unit for Cookies with 10 Opportunities for Defect per Cookie

5.

U Chart: monitor nominal scaled Defects per Units



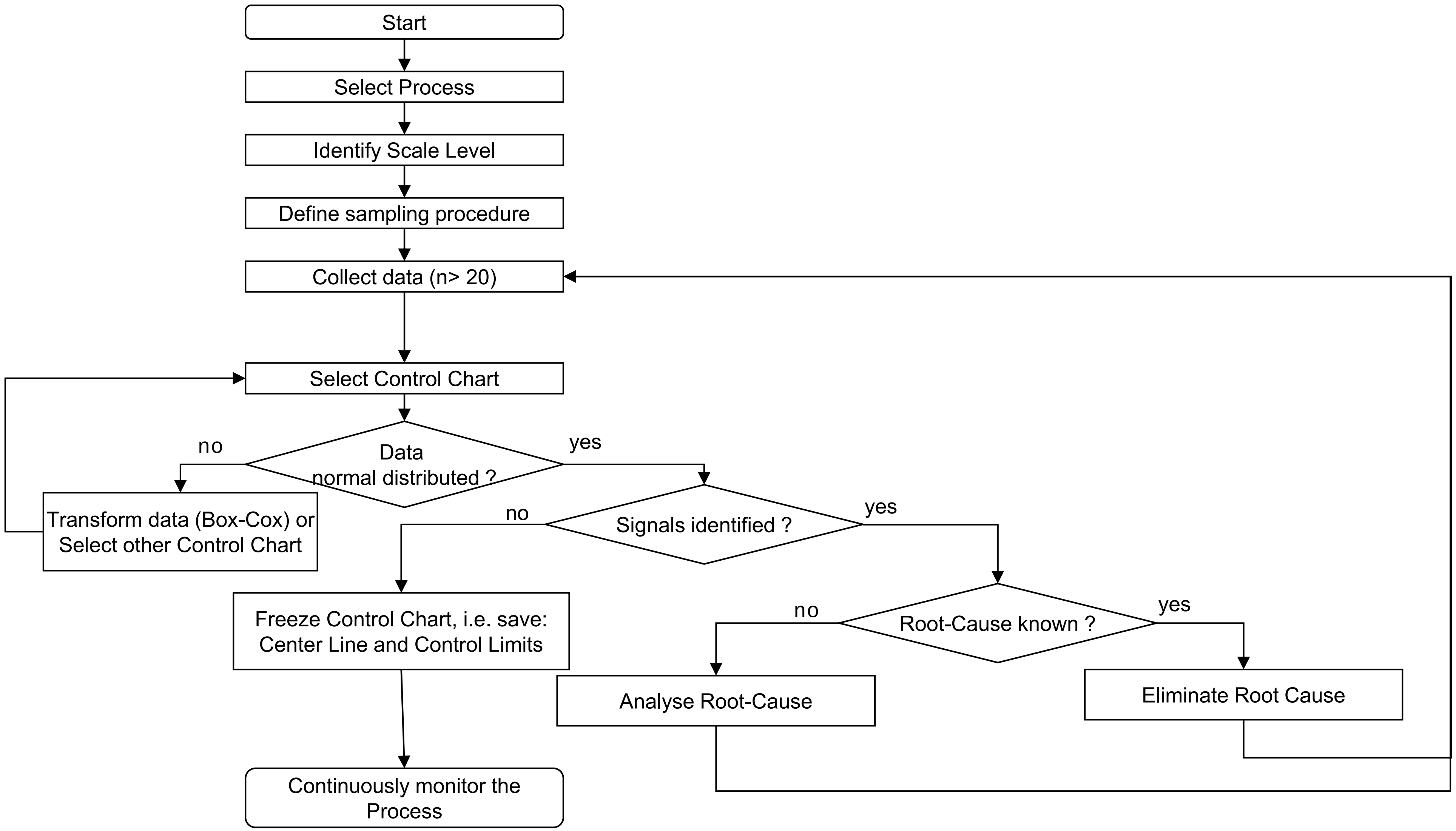
Results: Charts	
a. u-chart	Parameter for the Center of the average Defects per Unit
u-bar	
USL/ LSL	
Variation chart	not defined

Result: Comments (1/2)	
c.	Process stability statement: - displays the percentage of values out of control - the coloured bar indicates, whether the Process is under (yes) or out of control (no)
d.	Comments: Summary and comments about results

Result: Comments (2/2)	
e.	The dots in both Charts represent the single Products/ Services with their Defects in their Opportunities for Defects If the Charts are blacked out with a too large number of dots, then narrow the time interval
f.	note: The stable Upper Control Limit results from the constant number of Opportunities; with a variable number of Opportunities the Upper Control Limit becomes variable respective

Example: monitor the Defects per Unit for Cookies with 10 Opportunities for Defect per Cookie

Procedure to implement a Control Chart …



… up to ist continuous monitoring

Statistical Tests

Introduction

Observations of the reality are modelled as Hypothesis about Relationships or Differences …

format of the statistical examination

modeling of the observations		H_0	H_A
Relationship	**modeled observation**		
		There is no relationship between x and Y according to: - If (x), then (Y). - The (x), the (Y).	There is a relationship between x and Y according to: - If (x), then (Y). - The (x), the (Y).
	Example		
		There is a/no relationship between x and Y according to: - If the temperature of oven is (too) high (x), then the Cookie is burnt (Y). - The higher the temperature of the oven (x), the darker the Cookie (Y).	
	statistical formulation		
		$r_{xY} = 0$	$r_{xY} \neq 0$
Difference	**modeled observation**		
		There is no Difference - in the degree of: Y - between the Levels of: x (xi, xj, ...)	There is a Difference - in the degree of: Y - between the Levels of: x (xi, xj, ...)
	Example		
		There is a/ no Difference - in: the weight of Cookies (Y) - between: Types of Cookies (x) (e.g. Vanilla vs. Chocolate vs. ...)	
	statistical formulation		
		$Y_i = Y_j$	$Y_i \neq Y_j$

… and formally split into the Hypothesis H_0 vs. H_A for their statistical examination

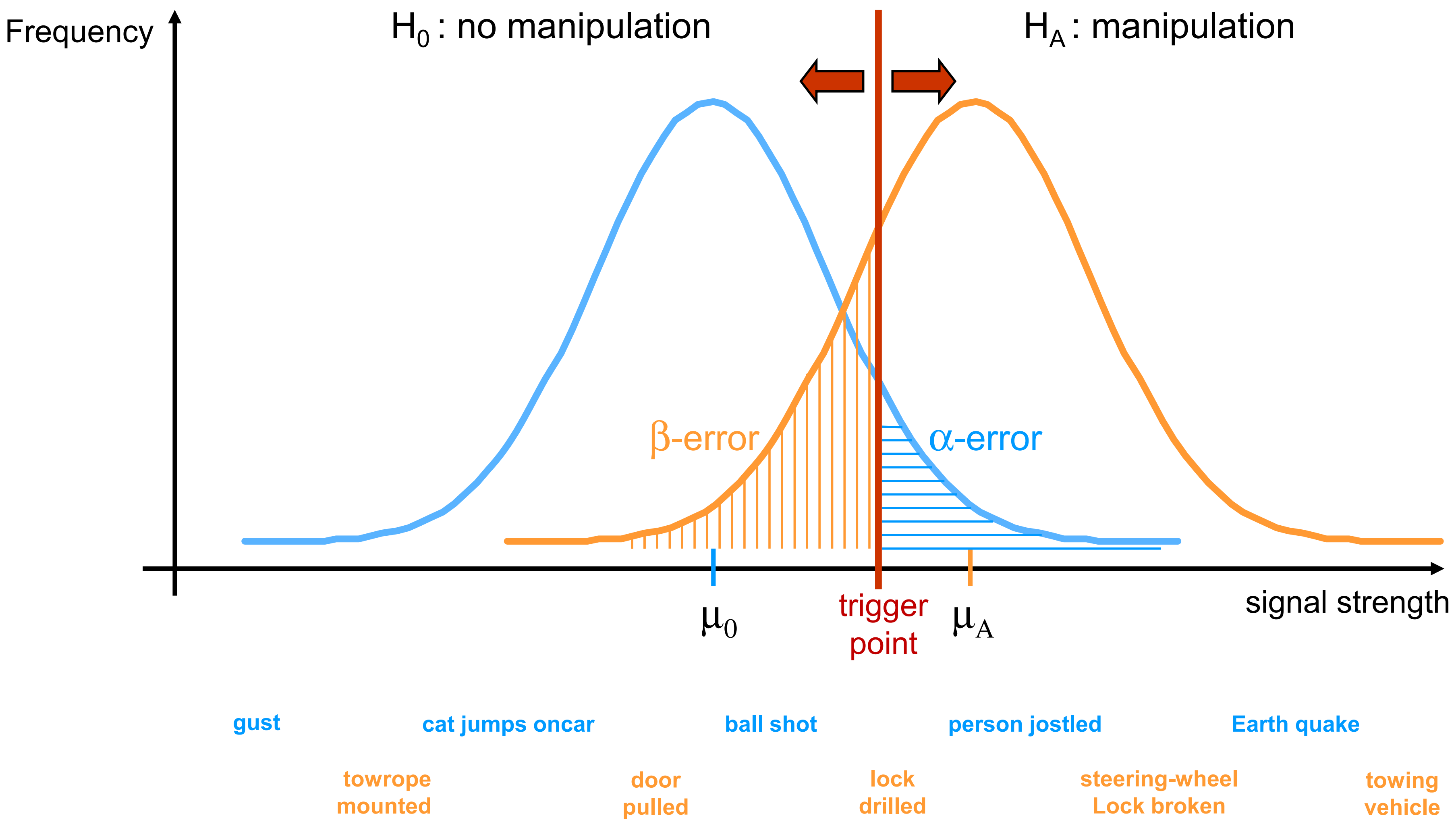
Selection of the appropriate Statistical Test for the examination of Hypothesis …

Y (columns) / **x** (rows)

x \ Y	Data in 2 Levels (Nominal-Scale)	Data in > 2 Levels (Nominal-Scale)	Data Rank Ordered (Ordinal-Scale)	Data discrete or continuous (Cardinal-Scale)
Data in 2 Levels (Nominal-Scale)	Relationship Hypothesis Chi-Square-Test	Relationship Hypothesis Chi-Square-Test	Difference Hypothesis Wilcoxon-Mann-Whitney-Test	Difference Hypothesis t-Test
Data in > 2 Levels (Nominal-Scale)	Relationship Hypothesis Chi-Square-Test	Relationship Hypothesis Chi-Square-Test	Difference Hypothesis Kruskal-Wallis-Test	Difference Hypothesis ANOVA
Data Rank Ordered (Ordinal-Scale)	Relationship Hypothesis Binary-Logistic-Regression	Relationship Hypothesis Nominal-Logistic-Regression	Relationship Hypothesis Rank Correlation (Spearman) / Ordinal-Logistic-Regression	Relationship Hypothesis Rank Correlation (Spearman)
Data discrete or continuous (Cardinal-Scale)	Relationship Hypothesis Binary-Logistic-Regression	Relationship Hypothesis Nominal-Logistic-Regression	Relationship Hypothesis Rank Correlation (Spearman) / Ordinal-Logistic-Regression	Relationship Hypothesis Product-Moment-Correlation (Pearson) / General Regression

… in the dependence of the Scale Level of the involved Variables (x and Y)

Example: right and wrong decisions of a car alarm



Every decisions has at least two outcomes and every outcome can be right and wrong

Decisions and Risks in Testing Statistical Hypothesis …

True is in reality/ in the population:

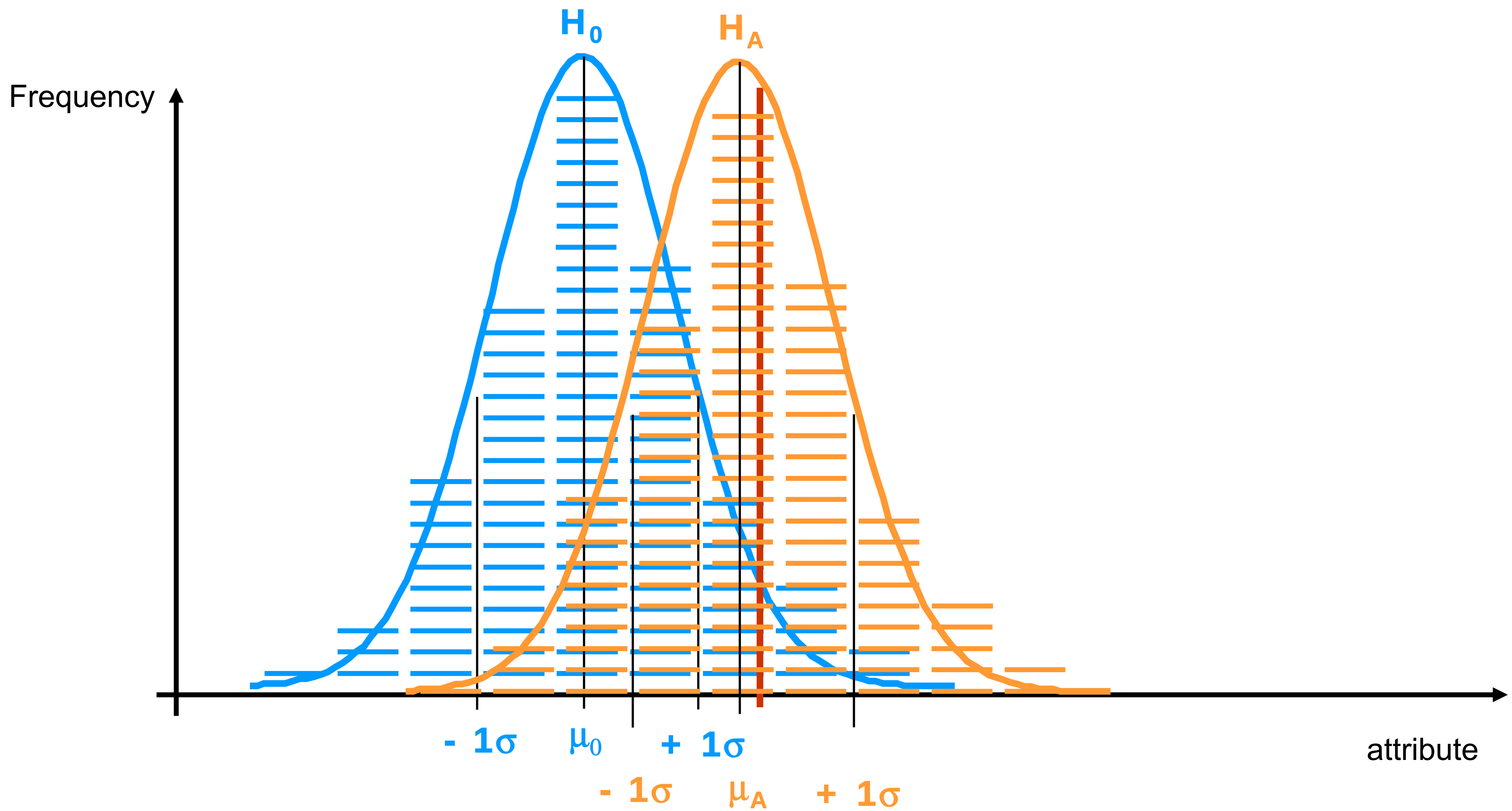
Decision based on results of a Statistical Test:		H0	HA
	H0	correct decision	wrong decision
		correct rejection	missed signal
		1-alpha	**beta (type 2 error)**
		Specifiity of the test	Sensitivity error
	HA	wrong decision	correct decision
		false Alarm	hit
		alpha (type 1 error)	**1-beta**
		Specification error	Sensitivity of the test

… then consider these Significance Levels:

If a decision should support:		**alpha (type 1 error)**	**beta (type 2 error)**
	H0	20%	5%
	HA (typical case)	5%	20%

… and recommended Significance Levels

The Decisions about Hypothesis depend on …



… the variability of attributes, effect size, alpha-error, beta-error and sample size

Every Statistical Test supports the Decision between H0 and HA …

The purpose of each Statistical Test is to support the decision between H0 and HA, i.e. to support the decision whether a Difference or a Relationship in the Population is given or not.

This Decision depends on several factors:
- the “true” Difference/ Relationship between the attributes in the Population
- the “true” dispersion of the Difference/ Relationship between the attributes in the Population
- the degree of Difference/ Relationship that has a practical relevance/ value
- the dispersion of Difference/ Relationship in samples
- the alpha-error of the Decision to accept “false alarms”
- the beta-error of the Decision to “miss signals”
- the sample size.

These factors mutually influence each other and we have to specify the optimal tradeoff for our decision. If the “rule of the thumb” values for the different purposes are accepted for the
- alpha-error (1%, 5%, 10%) and
- beta-error (20%) and its inverse value (Power= 1-beta= 80%)

then the tradeoff remains between Sample Size and the degree of Difference/ Relationship that has a practical relevance/ value. Thus in practice, the Sample Size is determined by the tradeoff between:

- the expense of data collection, and
- the size of Difference/ Relationship we want to identify - due to its Practical Relevance.

… based on Statistical Significance and Practical Relevance of ist results

Basic ideas and concept of all Statistical Tests (1/2)

Calculate Test Statistic: Chi-Square (χ^2):

Chi-Square Tests evaluate the Observed Frequencies of at least two categories in relation to Expected Frequecies in these at least two categories. The Expected Frequecies assume a H0 with uniform distributions in the categories.

Chi-Square Test Statistic: $$\chi^2 = \sum_{i=1}^{k} \frac{(O_i - E_i)^2}{E_i}$$

If the Difference between O_bserved and E_xpected frequencies is small, than $(O-E)^2$ is small, Chi-Square is small, meaning:
- H0 confirmed

If the Difference between O_bserved and E_xpected frequencies is high, than $(O-E)^2$ is high, Chi-Square is high, meaning:
- H0 to be rejected

The larger the Difference between:
O_bserved and E_xpected,
i.e. the larger the Chi-Square Value,
the lower the probability, that the O_bserved frequencies are collected from a Population, where H0 is true.

Preferences for Cookie-Types	O_bserved frequencies	E_xpected Frequencies	(O-E)2	(O-E)2/ E (Contributions to Chi2)
Vanilla	30	25	25	1
Chocolate	25	25	0	0
Cocos	40	25	225	9
Muffins	5	25	400	16
Summe	100	100	650	Chi-Square= 26

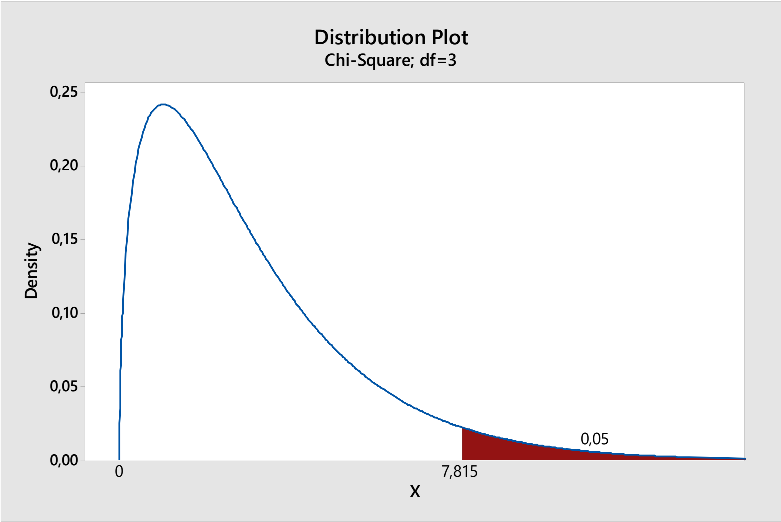
(1. Develop a test statistic) → 2. Calculate the value of the test statistic and …

Basic ideas and concept of all Statistical Tests (2/2)

Sampling Distribution of Chi-Square:

The Sampling Distribution of the Chi-Square Test Statistic follows the Chi-Square-Distribution, with degrees of freedom df= i-1
(Number of different categories (Cookie-Types) -1; df= 4- 1= 3).

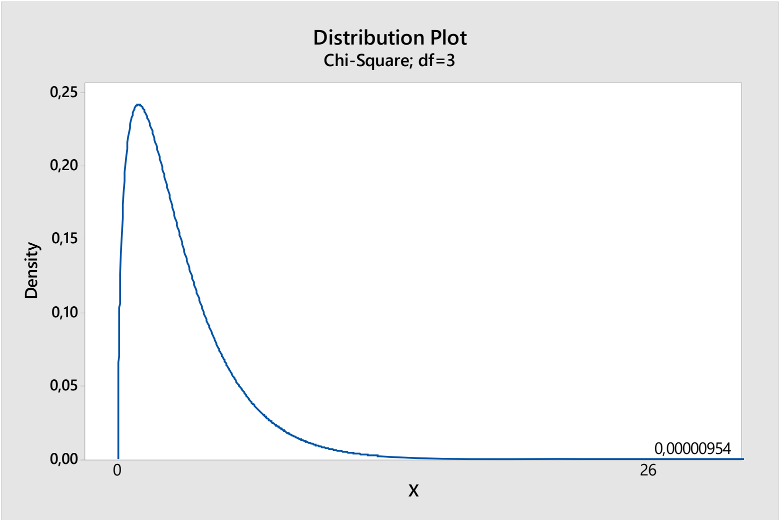
H0 is rejected, if the value of Chi-Square is so high, that its probability p ≤ alpha



Example for Cookies:
Chi-Square= 26,
df= 3
p= 0,00000954.

With alpha= 5% the result is significant (p<= alpha):

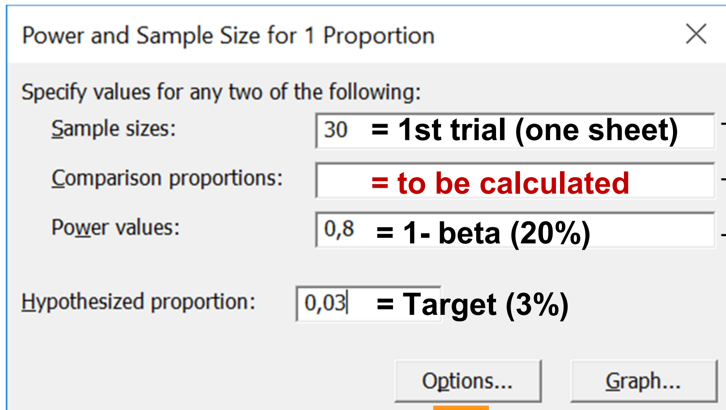
H0: rejected
HA: There is a Difference in: the Preference (Y) between: Cookie-Types.



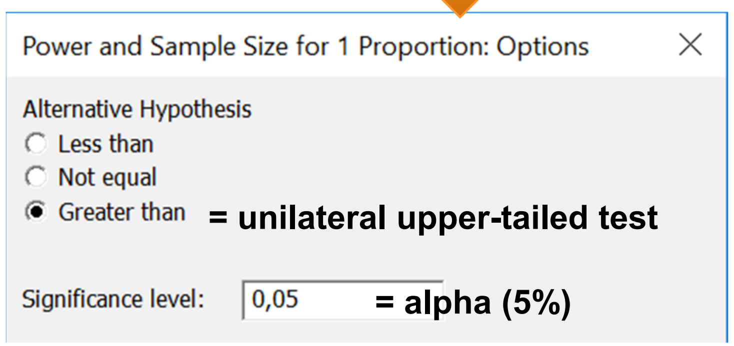
3. Calculate the probability of the value of the Test Statistic and 4. compare it with alpha

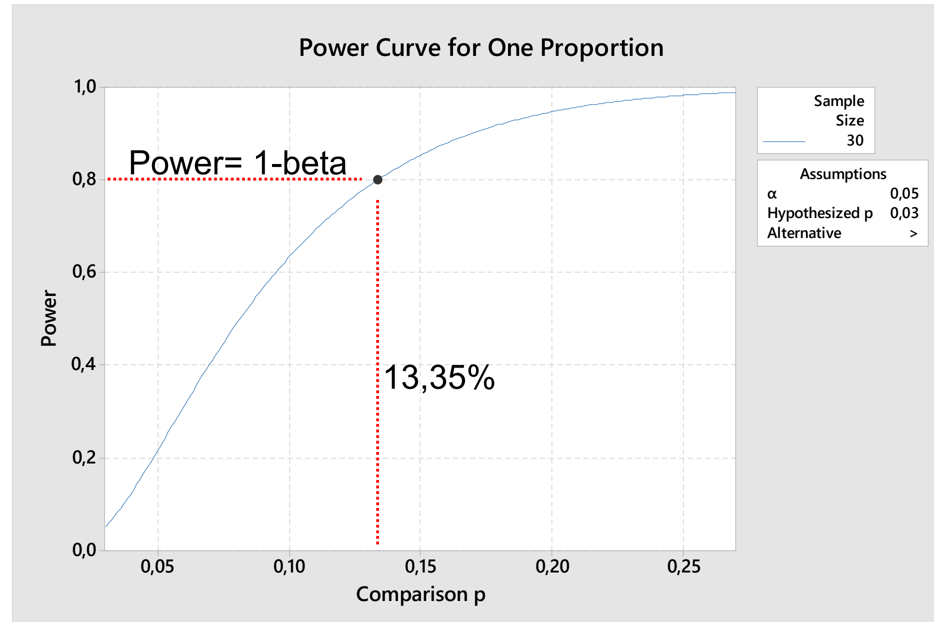
The balanced optimum between Sample Size and detectable Effect (Difference/ Relationship) …

If you want to evaluate, whether the Quality of the Cookies meets the target of: < 3% (ca. 1 defect/ Sheet) and if you could easily collect data about the status of 30 produced Cookies of one sheet, then you could calculate the detectable % of Defects, which are necessary to indicate that the target has been exceeded:



2 of these 3 fields need to be specified





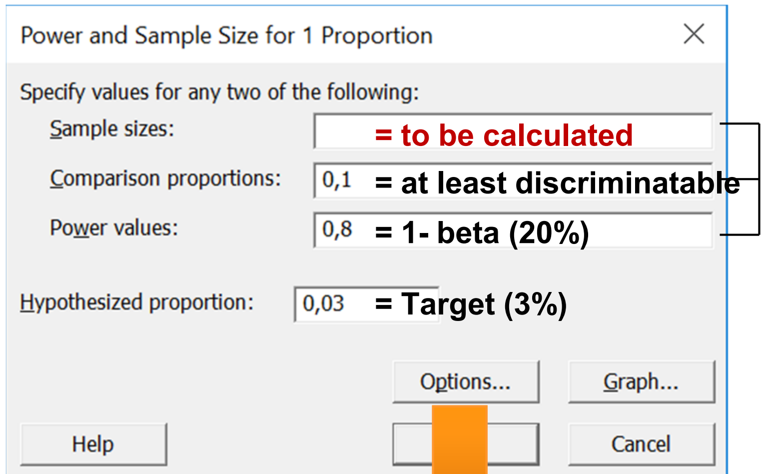

```
Testing p = 0,03 (versus > 0,03); α = 0,05

Sample
  Size  Power  Comparison p
    30    0,8      0,133488
```

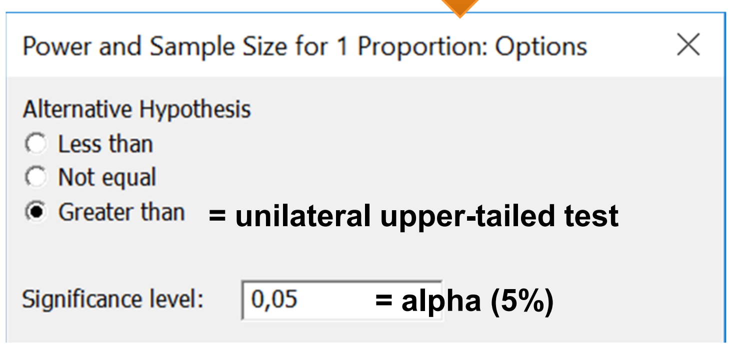
… can be identified by starting with a 1st idea of an affordable sample size

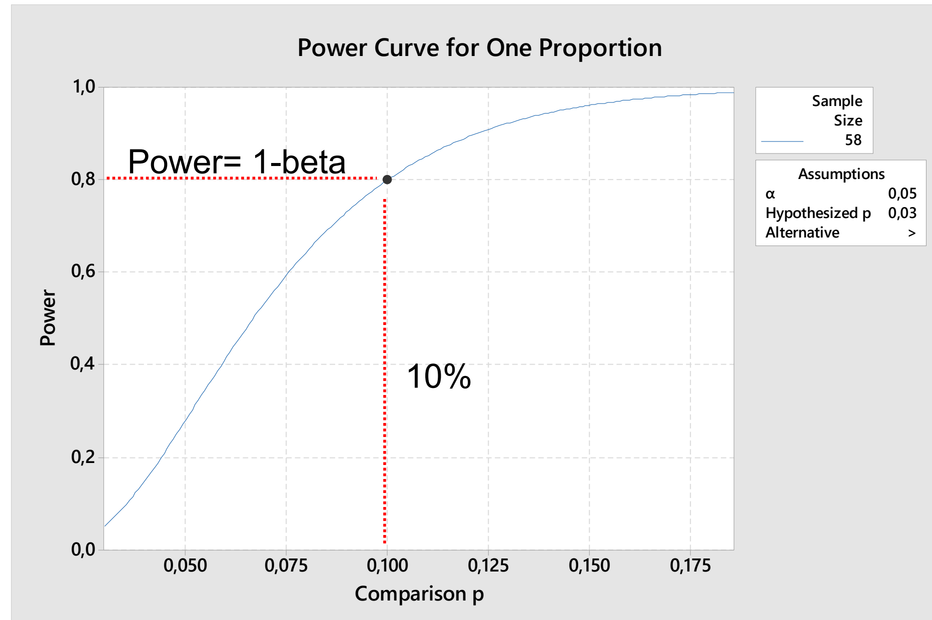
If the Difference/ Relationship, that can at least be detected is too large for your purposes …

If you promised your Customer to refund the money, if there are: >= 10% Defects, then you could recalculate, how big the sample needs to be, to detect at least 10% of Defects, if they are given in the Population:



2 of these 3 fields need to be specified






```
Testing p = 0,03 (versus > 0,03); α = 0,05

                 Sample  Target
Comparison p       Size   Power  Actual Power
         0,1         58     0,8      0,800025
```

… then start a 2nd trial with the Difference/ Relationship which at least must be detected

Statistical Tests

Details

Overview of Statistical Tests in the Mintab Assistant

#	Test	Purpose	Hypothesis	Y	Scale Level	x	Scale Level	note ...	Alternative in Minitab Stat Menu
a.1	**1-Sample % Defective**	Compare the percentual amount of a variable (Y_Sample) with a target value (Y_Target)	Difference	1	nominal (counted -> discrete cardinal)	./.	./.	Input: single %-value (not a data column)	Stat/ Basic Statistics/ 1 Proportion
a.2	**Chi-Square Goodness-of-Fit**	Compare the relative frequencies of the categories of a variable (Y), in relation to a) their expected values or b) specific target values (YT)	Difference	1	nominal (counted -> discrete cardinal)	./.	./.	Input: enter values in Table or get Data from Worksheet	Stat > Tables > Chi-Square Goodness-of-Fit Test (One Variable)
a.3	**1-Sample t-Test**	Compare the Mean of a variable (Y) with a target value (YT)	Difference	1	cardinal	./.	./.	Y-Variable: normal distributed; Sample Size N: > 20	Stat/ Basic Statistics/ 1-Sample t
b.4	**2-Sample t-Test**	Compare the Means of two independent variables with each other (Y1 vs Y2)	Difference	1	cardinal	1	nominal	Y-Variable: normal distributed; Sample Size N: > 20	Stat/ Basic Statistics/ 2-Sample t
b.5	**Paired t-Test**	Compare the Means of two dependent/ matched variables with each other (Ya vs. Ya´)	Difference	1	cardinal	1	nominal	Y-Variable: normal distributed; Sample Size N: > 20	Stat/ Basic Statistics/ Paired t
b.6	**2-Sample Standard Deviation**	Compare the Standard Deviations of two independent variables (Y1 vs. Y2)	Difference	1	cardinal	1	nominal	Y-Variable: normal distributed; Sample Size N: > 20	Stat/ Basic Statistics/ 2 Variances
c.7	**One-Way ANOVA**	Compare the differences in the Means of a dependent variable (Y) in respect to factorial scaled independent variable (x)	Difference	1	cardinal	1	nominal / ordinal	Y-Variable: normal distributed; Sample Size N: > 20; N of all Factorial Levels must be the same	Stat/ ANOVA/ One-Way or other
c.8	**Chi-Square % Defective**	Compare the percentual amounts Y of different factor levels of one attribute x (e.g. % defective vs. not; % sold vs. not)	Difference	1	nominal (counted > discrete cardinal)	1	nominal	Test does not have to be about defects, but about interesting portions in any other attribute X. - Number of Factor Levels of Xi can vary from: 3 - 12.	
c.9	**Chi-Square Test for Association**	Compare the percentual amounts of Y in respect to the factorial levels of 2 categorial Variables (Xi, Xj)	Difference	1	nominal (counted > discrete cardinal)	2	nominal	Number of Factor Levels for Xi and Xj can vary from: 3 - 6.	Stat/ Tables/ Chi-Square-Test for Association
d.10	**Regression**	Analyse the relationship between (multiple) x and Y	Relationship	1	cardinal	n	cardinal	X- and Y-Variables: normal distributed; Sample Size N: > 15	Stat/ Regression

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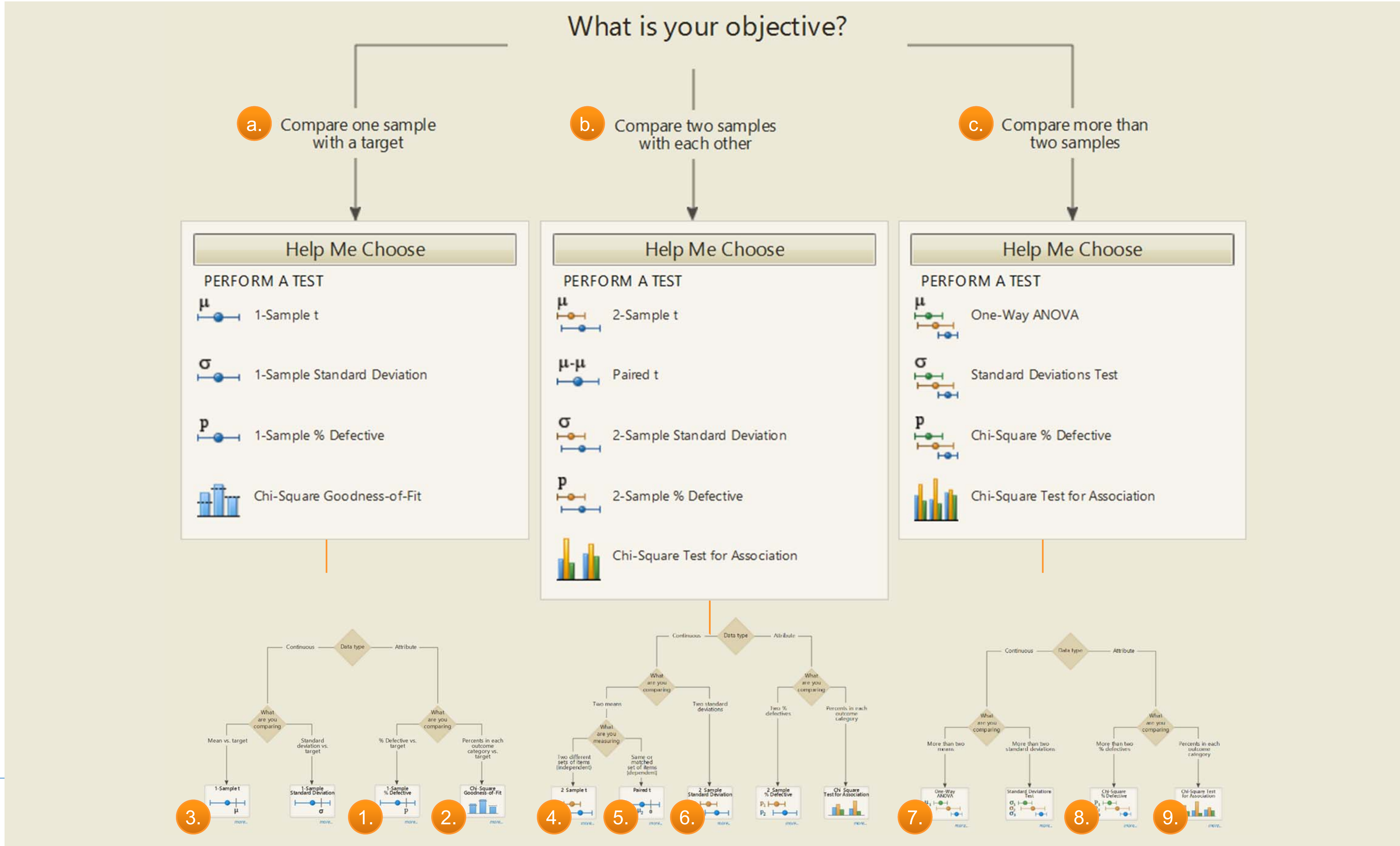
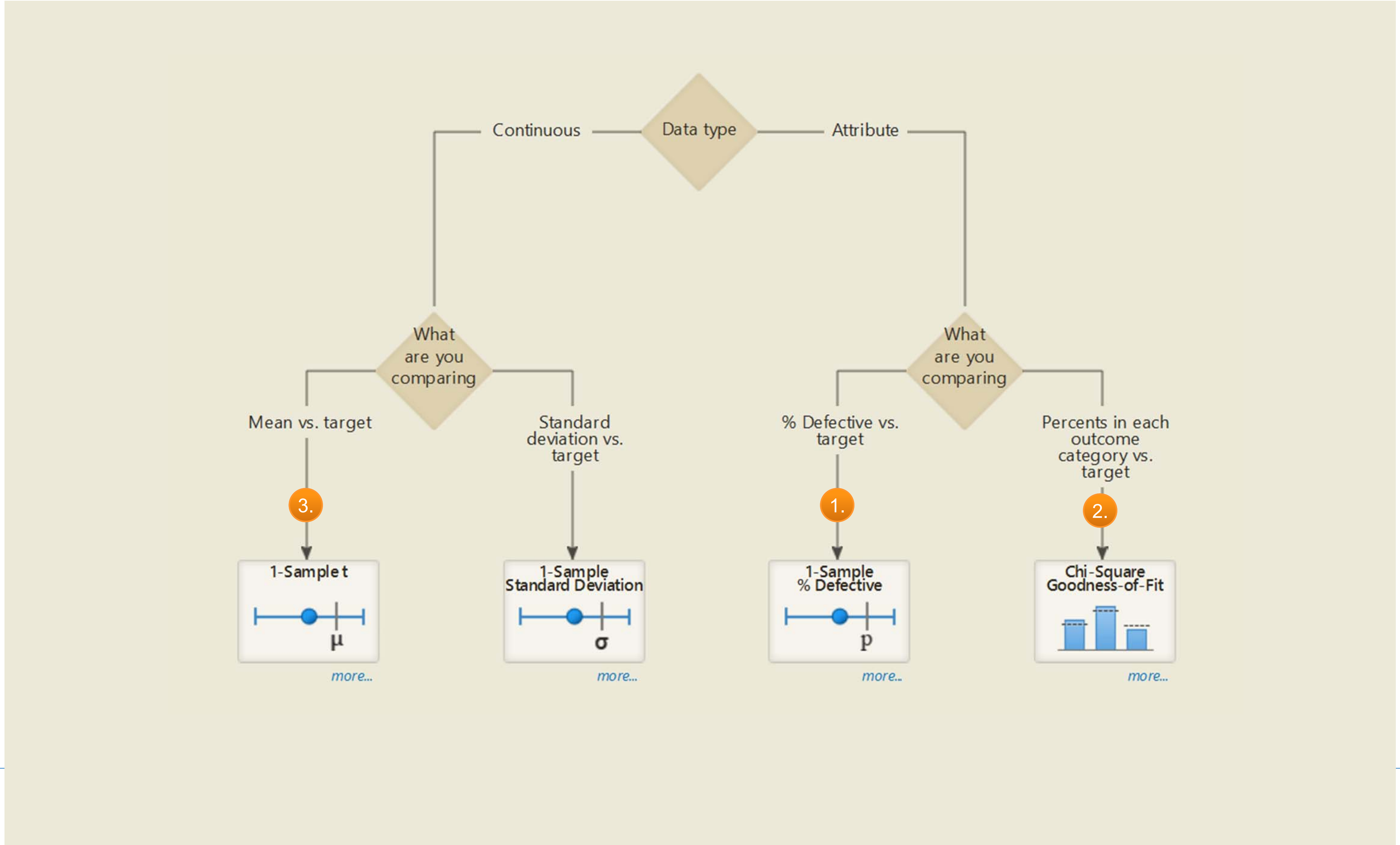
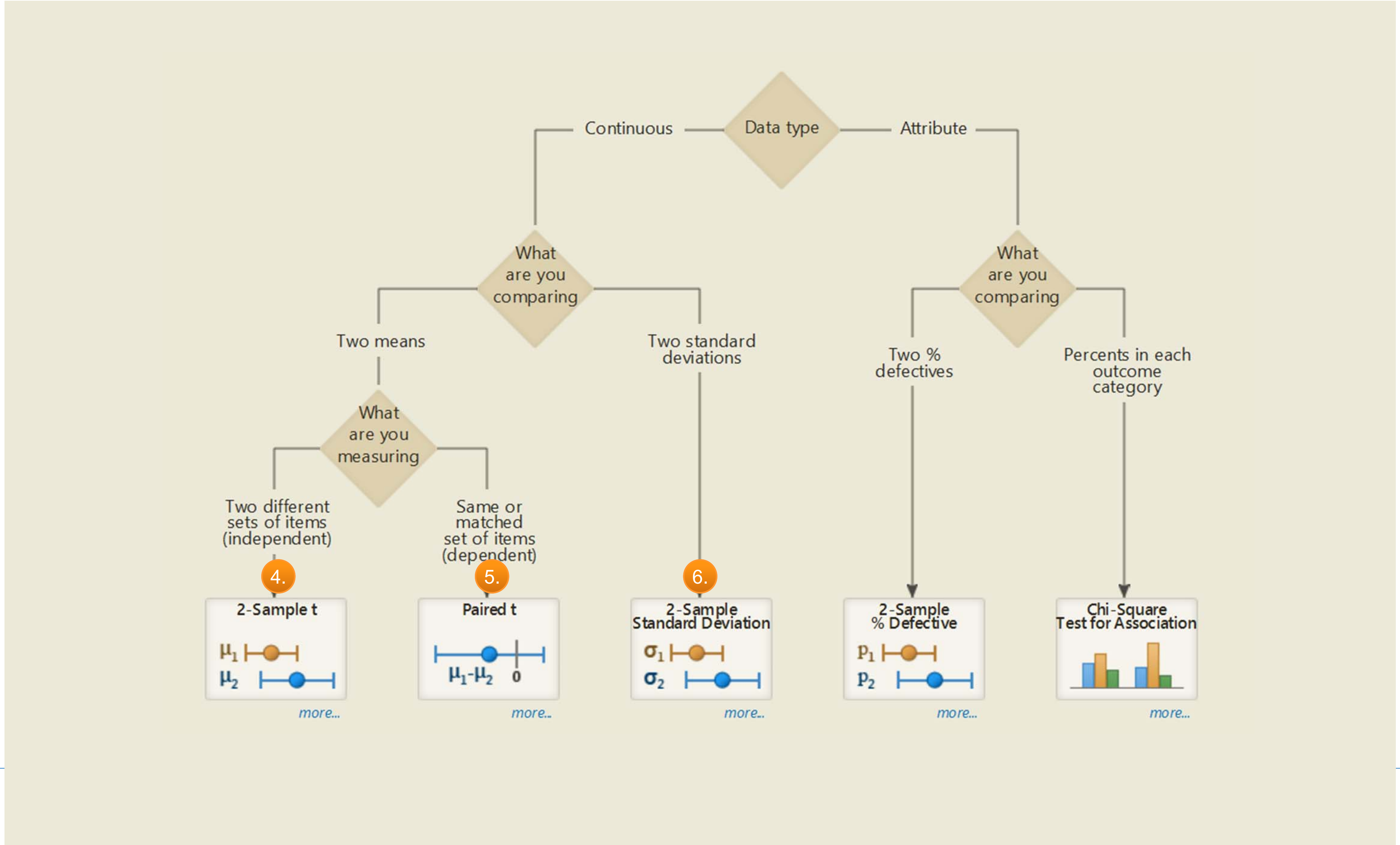
Selection of the suitable Statistical Test: Overview



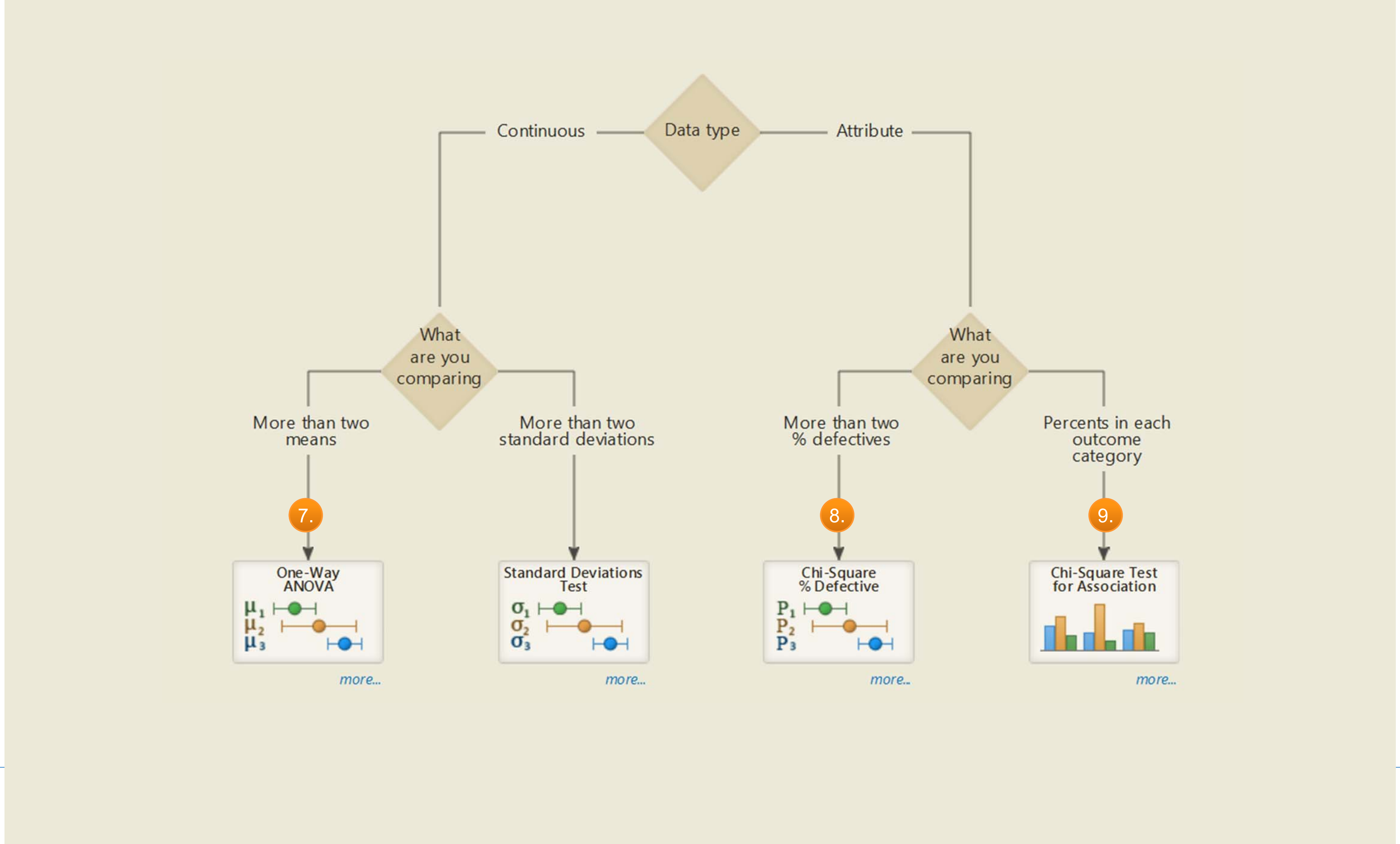
a. ***Compare one sample with a target***



b. *Compare two samples with each other*



c. *Compare more than two samples*



1.

1-Sample % Defective: compare a percentual amount with a target value

Test

1-Sample % Defective

Purpose

Compare the percentual amount of a variable (Y_Sample) with a target value (Y_Target)

Hypothesis

Difference	There is no/ a Difference in: the percentual amount of Defectives between: Y_Sample and Y_Target

Example

The portion of errors compared to a target value

The votes for a specific political party compared with the electoral threshold (e.g. 5%)

The portion of defect Cookies in one package

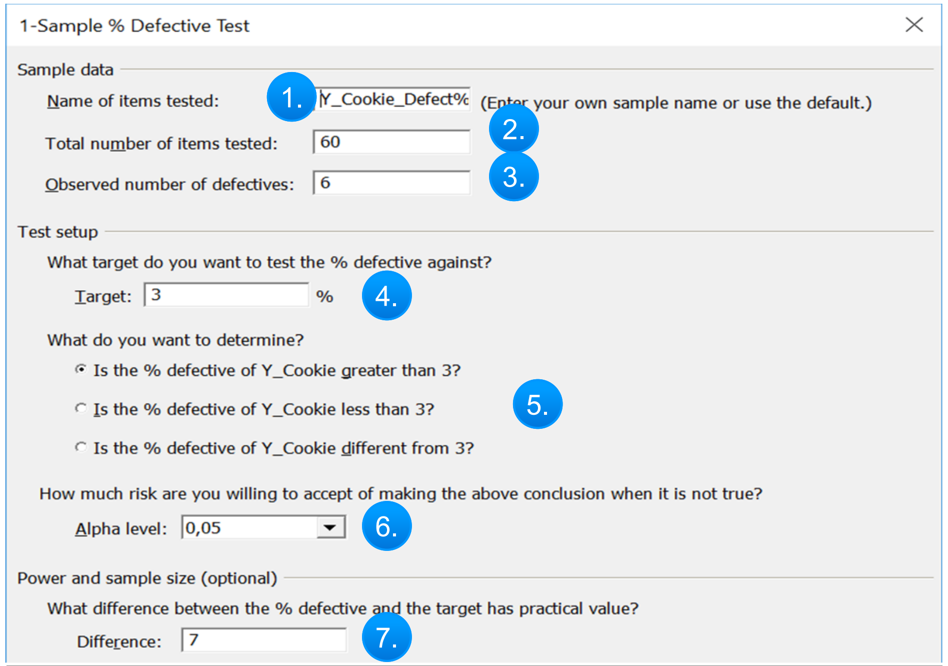
Y	**Scale Level**
1	nominal (counted -> discrete cardinal)
x	**Scale Level**
./.	./.

Alternative in Minitab Stat Menu

Stat/ Basic Statistics/ 1 Proportion

note …

Input: single %-value (not a data column)

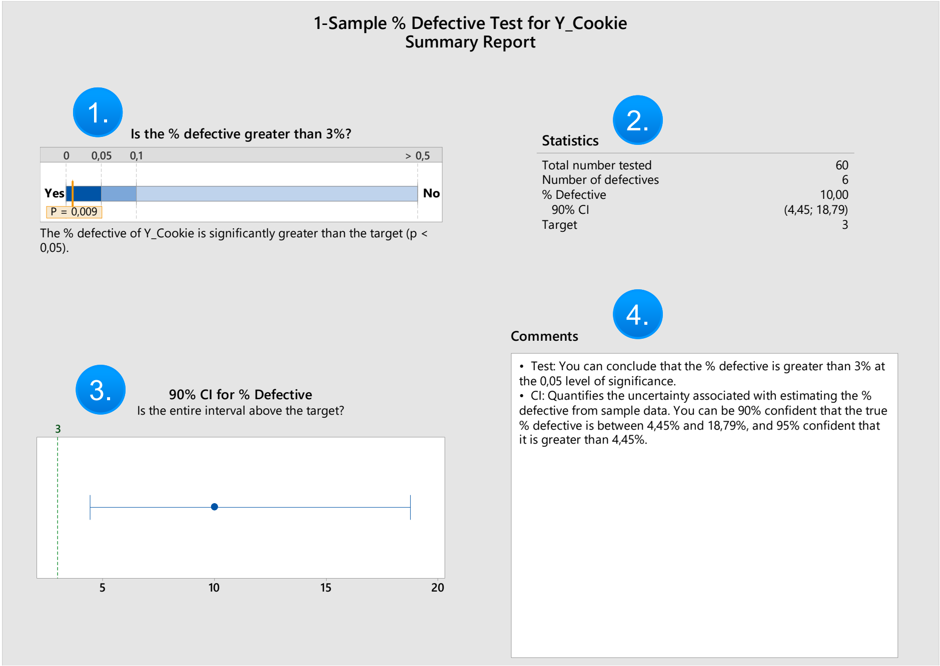


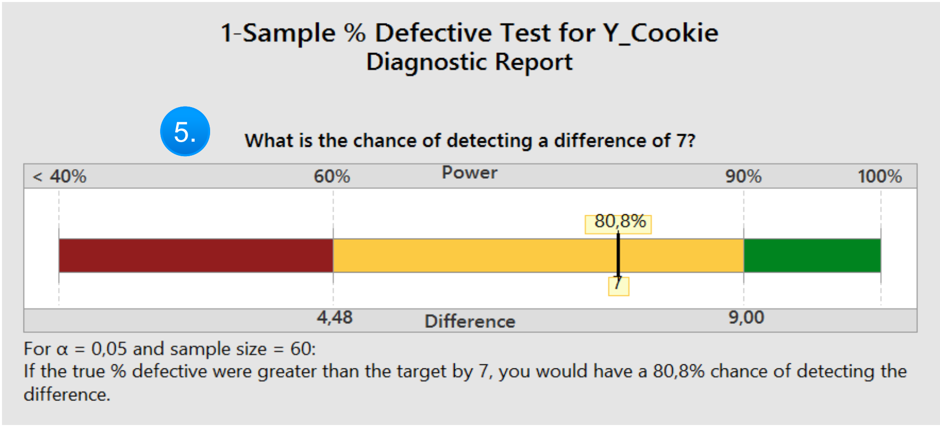
Dialog	
1. Name	Enter the name of your variable Y
2. No. of Items tested	Data input: all units of the sample (integer)
3. Defective Units	Data input: defective Units of the sample (integer)
4. Target	Maximum-/ Target value or proportion of errors that is acceptable (integer as %)
5. Direction of Test	% defective > target value (one-sided significance test)
	% defective < target value (one-sided significance test)
	% defective ≠ target value (two-sided significance test)
6. Alpha-Level	Significance level for the decision of the Test
7. Effect Size	Critical difference between %-defectives and the %-target values, that should be at least discriminated, because of its practical value for the analysis

Example: compare the % defective Cookies baked per day with a target

1.

1-Sample % Defective: compare a percentual amount with a target value





Results	
1. Significance Test	Hypothesis: Is the % defective greater than target%?
	The bar of the chart indicates: - orange line: actual p-value of the Significance Test - dark blue sector: 0% > alpha <= 5% - light blue sector: 5% > alpha <= 10%
	accept H0 , if p> alpha, e.g. There are no differences accept HA, if p<= alpha, e.g. There is a difference
2. Values to categories	Statistics with description of Sample, Parameter and Confidence Interval
3. Chart	Details for the significance test: Interval Chart with confidence intervals for the Parameter of the variable, that includes the value of the Target (H0) or does not include (HA)
4. Comments	Summary and comments about results
5. Power & Sample Size	Power of the Sample Size to detects the practical relevant difference (if it is present in the population)
	Example: the critical, to be identified difference of 7% if present (3% Defects:= Target value, 10% Defects:= Customer gets money back -> Difference =7%), can be identified with the probability of 80,8% with the Sample Size of 60 (Rule of the thumb: necessary Power= 80% = 1-beta)
6. Power & Sample Size	- Relations between Power and Sample Size for different levels of Power - actual Power and Sample Size of the Test

6. **What sample size is required to detect a difference of 7?**

Sample Size	Power
26	60%
40	70%
58	80%
91	90%

Your Sample

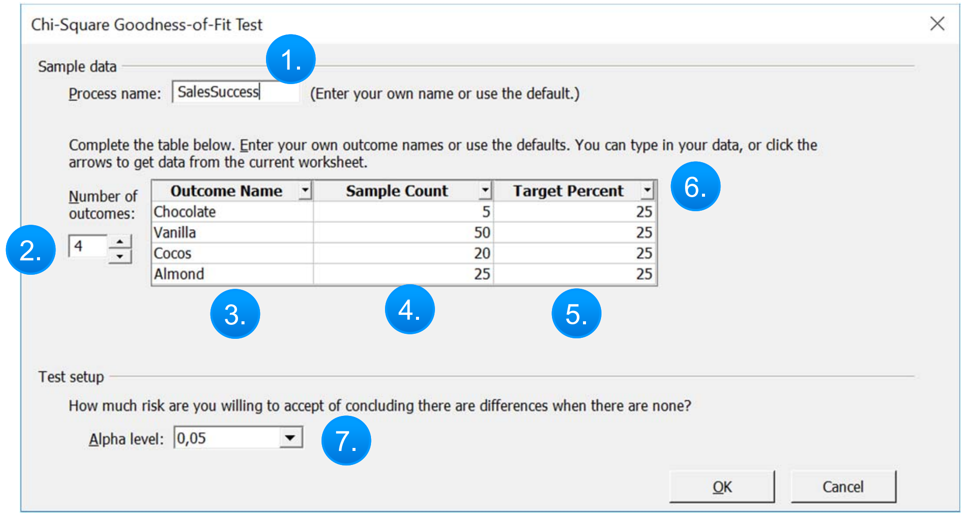
60	80,8

Example: compare the % defective Cookies baked per day with a target for defects

2.

Chi-Square Goodness-of-Fit Test: Compare actual with expected or target frequencies

Test	
Chi-Square Goodness-of-Fit	
Purpose	
Compare the relative frequencies of the categories of a variable (Y), in relation to a) their expected values or b) specific target values (YT)	
Hypothesis	
Difference	There is a/ no Difference in: percentual amounts of Y and Target-Value/ Expected-Value between: categories of Y
Example	
Comparison of the Portions of different Types of Errors with Targets	
Comparison of the Election Results of different Parties, based on a Sample, with the electoral threshold (e.g. 5%)	
Sales Success of different Cookie Types Portion of different Cookies in a package	
Y	**Scale Level**
1	nominal (counted -> discrete cardinal)
x	**Scale Level**
./.	./.
Alternative in Minitab Stat Menu	
Stat > Tables > Chi-Square Goodness-of-Fit Test (One Variable)	
note …	
Input: enter values in Table or get Data from Worksheet	



Dialog	
1. Name	Enter a name for this Analysis
2. No. of Outcomes	Number of different categories of the variable
3. Outcome Name	Names of the different categories of the variable Y
4. Sample Count	Number of cases in the specific categories
5. Target Percent	Number of cases or percentage of cases expected or Target-% for each category of the variable
6. Input Format	(enter values or get Data from Worksheet)
7. Alpha-Level	Significance level for the decision of the Test

Example: Sales Success of different Cookie Types

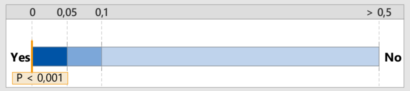
2.

Chi-Square Goodness-of-Fit Test: Compare actual with expected or target frequencies

1.

Chi-Square Goodness-of-Fit Test for SalesSuccess
Summary Report

Do the process and target percents differ?



The percents for SalesSuccess are significantly different from the target percents (p < 0,05).

2.a

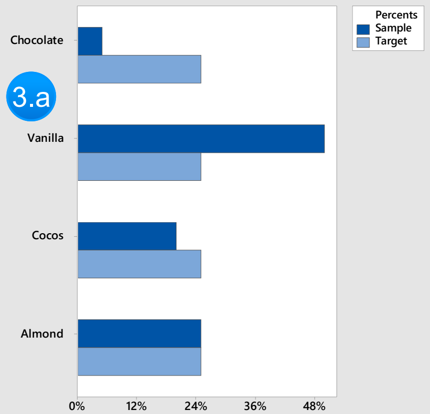
Outcome Table
Which outcomes differ from their target?

Outcome	Sample Percent	Target Percent	Differ
Chocolate	5,0	25	Lower
Vanilla	50,0	25	Higher
Cocos	20,0	25	No
Almond	25,0	25	No

Total count = 100

3.a

Outcome Comparison Chart
Compare the sample and target percents.



4.

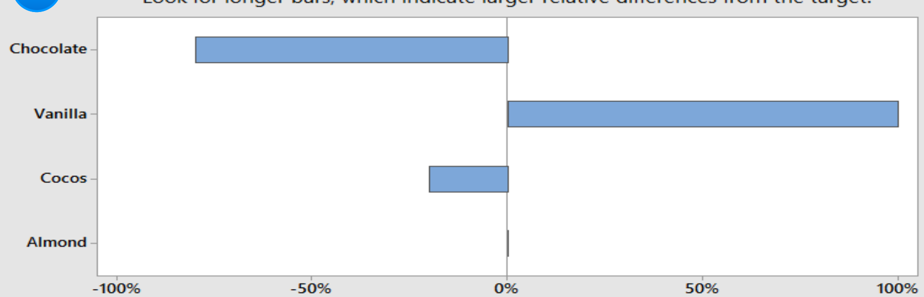
Comments

- Test: You can conclude that the process percents differ from the target percents at the 0,05 level of significance.
- Outcome Table: Shows which process percents differ from their target percent. Consider the size and direction of the difference to determine if it has practical implications.
- Outcome Comparison Chart: Compare the relative size of the sample and target percents for each outcome.

Results	
1. Significance Test	Hypothesis: Do the actual and target values of the Output differ (over the categories)?
	The bar of the chart indicates: - orange line: actual p-value of the Significance Test - dark blue sector: 0% > alpha <= 5% - light blue sector: 5% > alpha <= 10%
	accept H0 , if p> alpha, e.g. There are no differences accept HA, if p<= alpha, e.g. There is a difference
2. Statistics	a) Description of the Sample values, Target values and deviation for each category b) 95% Confidence Intervals (CI) for Sample Values (%)
3. Chart	a) Sample % and Target %
	b) Contribution to the Chi-Square Value (= Test Value) by Category , i.e. the higher the value/ longer the bar, the more contributes the effect of this category to the Significance of the result
4. Comments	Summary and comments about results

3.b

% Difference between Sample and Target Counts
Look for longer bars, which indicate larger relative differences from the target.



Positive differences indicate sample counts that are higher than the target.
Negative differences indicate sample counts that are lower than the target.

2.b

Outcome	Target Count	Target Percent	Sample Count	Sample Percent	Individual 95% CI
Chocolate	25	25	5	5,0	(0,7; 9,3)
Vanilla	25	25	50	50,0	(40,2; 59,8)
Cocos	25	25	20	20,0	(12,2; 27,8)
Almond	25	25	25	25,0	(16,5; 33,5)

To ensure validity of the test, the target count should be at least 1,25. To ensure validity of the intervals, the sample count should be at least 5.

Example: Sales Success of different Cookie Types

3.

1-Sample t-Test: Compare the Mean of a Variable with a Target Value

Test

1-Sample t-Test

Purpose

Compare the Mean of a variable (Y) with a target value (YT)

Hypothesis

Difference	There is a/ no Difference in: the level of Values (Y) between: Sample and Target

Example

Comparison of Cycle Times with a Target

Comparison of a Share Value with a Target

Comparison of Oven temperature with a Target

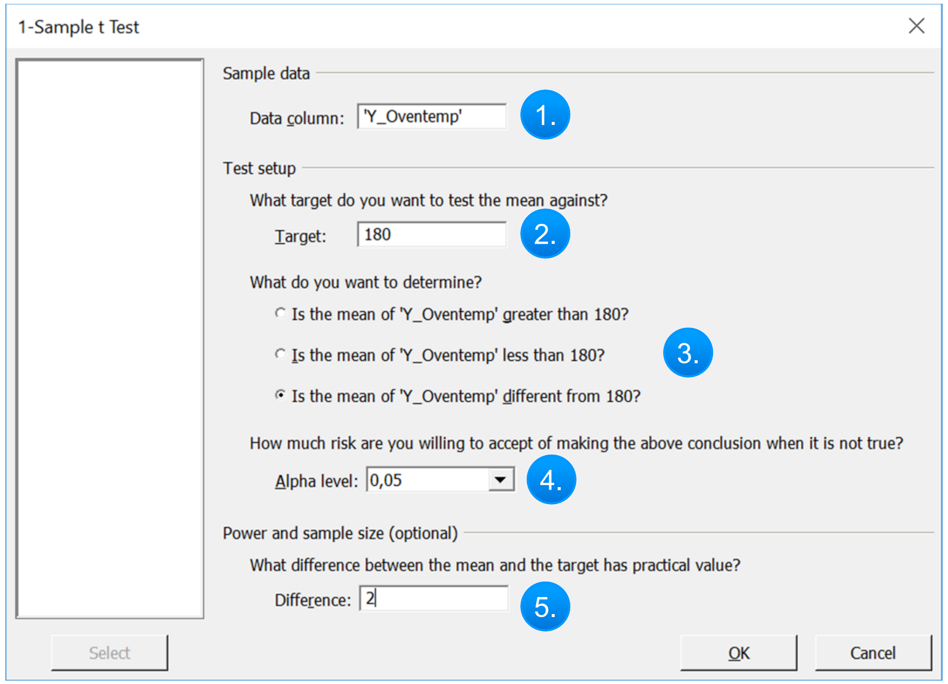
Y	**Scale Level**
1	cardinal
x	**Scale Level**
./.	./.

Alternative in Minitab Stat Menu

Stat/ Basic Statistics/ 1-Sample t

note …

Y-Variable: normal distributed; Sample Size N: > 20

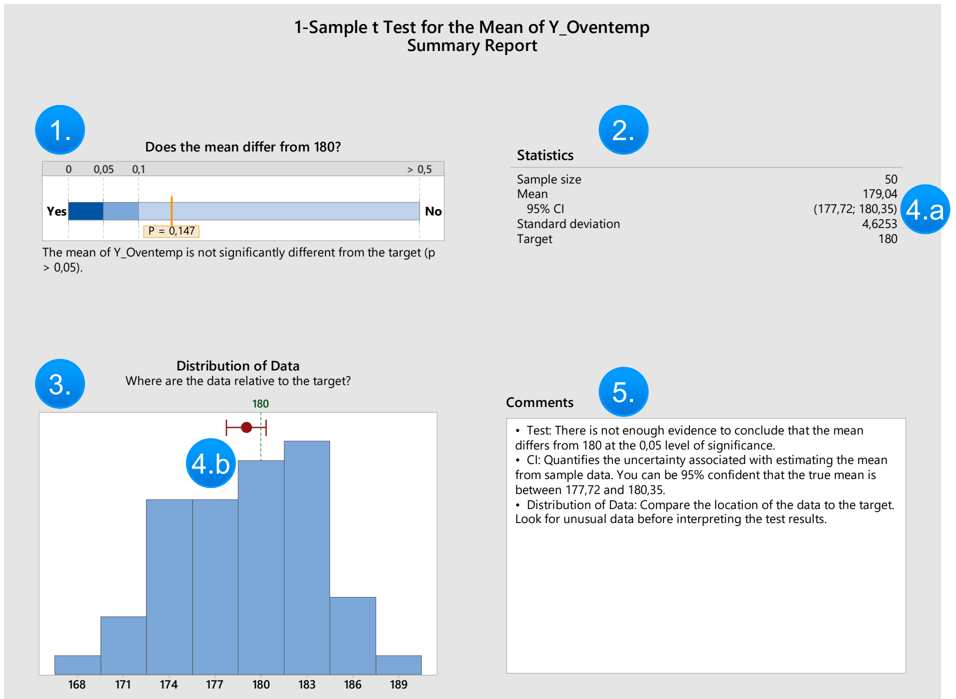


Dialog	
1. Data Column	Variable with the measured values
2. Target Value	Target Value, which should be compared with the Mean of the Variable
3. Type of Test	Mean of Sample > Target Value (one-sided significance test)
	Mean of Sample < Target Value (one-sided significance test)
	Mean of Sample ≠ Target Value (two-sided significance test)
4. Alpha-Level	Significance level for the decision of the Test
5. Power	Critical Difference between Sample and Target, that needs to be at least discriminable, if given

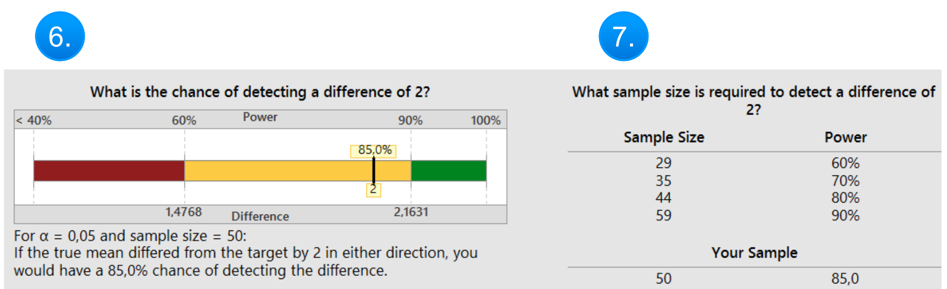
Example: Comparison of the Oventemperature with a Target given in a recipe

3.

1-Sample t-Test: Compare the Mean of a Variable with a Target Value



Results	
1. Significance Test	Does the Mean differ from 180 (Target Value)?
	The bar of the chart indicates: - orange line: actual p-value of the Significance Test - dark blue sector: 0% > alpha <= 5% - light blue sector: 5% > alpha <= 10%
	accept H0 , if p> alpha, e.g. There are no differences accept HA, if p<= alpha, e.g. There is a difference
2. Statistics	Description of the Sample, Mean, Standard Deviation and Target Value and the 95% Confidence Interval (CI) for the Mean
3. Histogram	Distribution of the Y-Variable
4. Target und CI	a) numerical values of the Mean and its Confidence Interval (CI)
	b) graphical representation of the Mean, its CI and the Target. If the target value is within the confidence interval, then the H0 is accepted, otherwise it is rejected.
5. Comments	Summary and comments about results
6. Power	Power (%) for critical Difference between Mean and Target, that needs at least to be identified
	The difference of >=2 can with the current sample size (N= 50) be identified with a probability of 85% (Rule of the Thumb: Power >= 80%)
7. Power	Power (%) for the to be identified critical difference between the Mean and the Target Value for different Sample Sizes (N)
	Example: the critical difference of 2 can be detected, if given, with a probability of 80%, if the Sample Size= 44



Example: Comparison of the Oventemperature with a Target given in a recipe

4.

2 Sample t-Test: Comparison of the Means of two (unmatched) Variables

Test

2-Sample t-Test

Purpose

Compare the Means of two independent variables with each other (Y1 vs Y2)

Hypothesis

Difference	There is a/ no Difference in: Mean between: Variables (Y_pre vs. Y_post)

Example

Comparison of the Revenue with the old vs new Product

Comparison of Cycle Time before vs. after the Project

Comparison of the weights: Chocolate Cookies vs. Vanilla Cookies

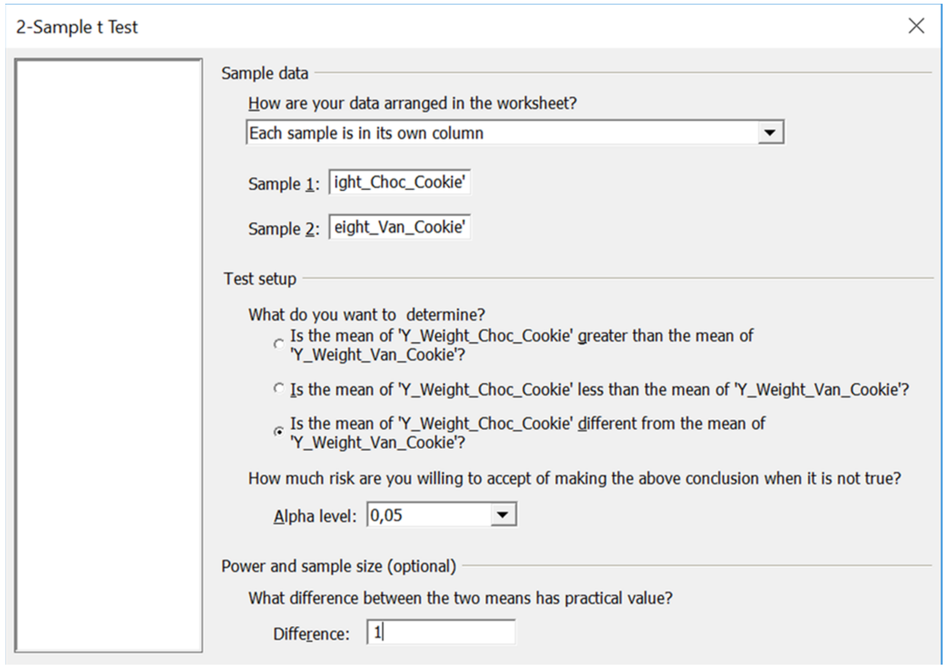
Y	Scale Level
1	cardinal
x	**Scale Level**
1	nominal

Alternative in Minitab Stat Menu

Stat/ Basic Statistics/ 2-Sample t

note …

Y-Variable: normal distributed; Sample Size N: > 20

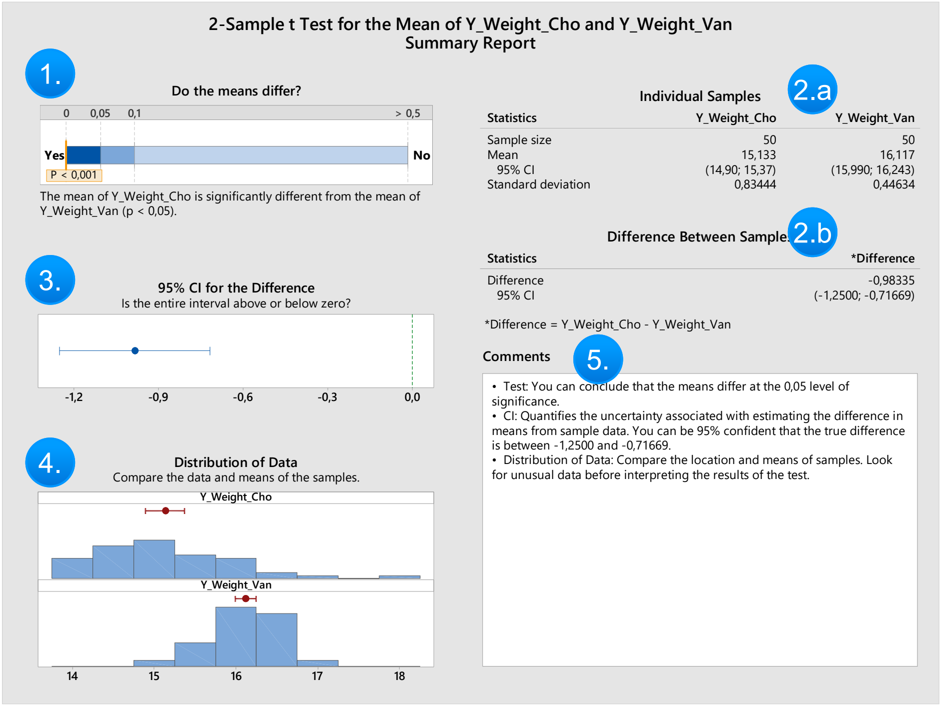


Dialog	
1. Sample Data	Arrangement of Data: see slide: Grouped data can be arranged in two alternative ways two cells with the measured data of both variables
2. Type of Test	Mean of Sample Y_1 > Mean of Sample Y_2 (one-sided significance test)
	Mean of Sample Y_1 < Mean of Sample Y_2 (one-sided significance test)
	Mean of Sample Y_1 ≠ Mean of Sample Y_2 (two-sided significance test)
3. Alpha-Level	Significance level for the test
4. Power	Critical Difference between Samples, that need to be at least discriminable, if given

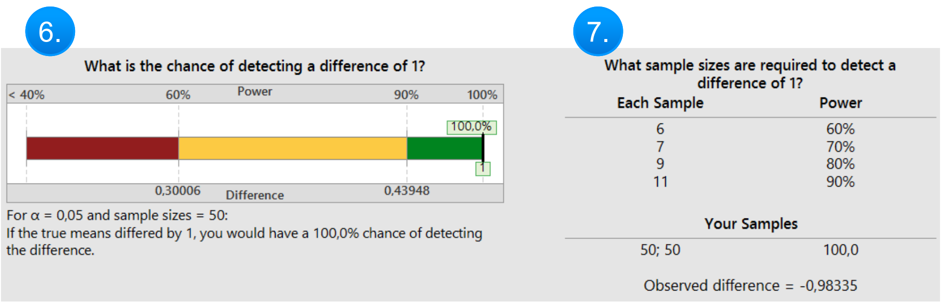
Example: Comparison of the Weight of two different Types of Cookies

4.

2 Sample t-Test: Comparison of the Means of two (unmatched) Variables



Results	
1. Significance Test	Do the Means differ?
	The bar of the chart indicates: - orange line: actual p-value of the Significance Test - dark blue sector: 0% > alpha <= 5% - light blue sector: 5% > alpha <= 10%
	accept H0 , if p> alpha, e.g. There are no differences accept HA, if p<= alpha, e.g. There is a difference
2. Statistics	a) Description of the Sample, Mean, Standard Deviation and the 95% Confidence Interval (CI) for the Means; b) difference between the Means, Confidence Interval of the difference
3. Interval-Plot	Confidence Interval of the difference in relation to the Value 0; (If CI contains 0, then H0)
4. Histogram	Distribution of Y_1 and Y_2
	Interval Plots for the Means of Y_1 and Y_2 and CI´s (If CI´s overlap, then H0)
5. Comments	Summary and comments about results
6. Power	Power (%) for critical Difference between Mean and Target, that needs at least to be identified
	The difference of >=1 can with the current sample size (N= 50) be identified with a probability of 100% (Rule of the Thumb: Power >= 80%)
7. Power	Power (%) for the to be identified critical difference between the Mean and the Target Value for different Sample Sizes (N)
	Example: the critical difference of 1 can be detected, if given, with a probability of 80%, if the Sample Size= 9

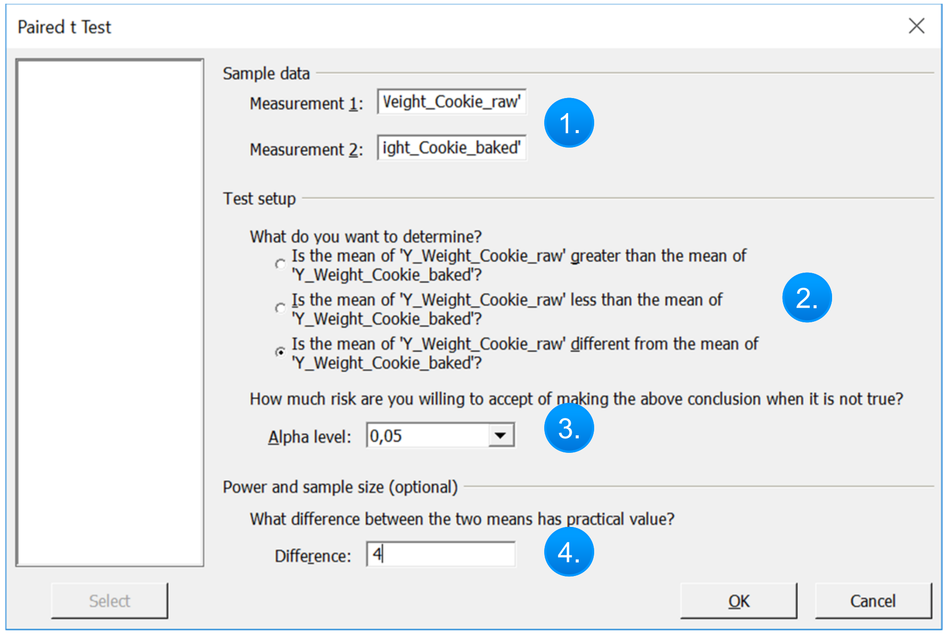


Example: Comparison of the Weight of two different Types of Cookies

5.

Paired t-Test: Compare the Means of two dependent/ matched Variables

Test	
Paired t-Test	
Purpose	
Compare the Means of two dependent/ matched variables with each other (Ya vs. Ya´)	
Hypothesis	
Difference	There is a/ no Difference in: Mean between: Variables (Ya vs. Ya´)
Example	
Compare the performance of Computers before vs. after Software update	
Comparison of Six Sigma Competence before vs. after Training	
Comparison of the weights of: Cookies_raw vs. Cookies_baked	
Y	**Scale Level**
1	cardinal
x	**Scale Level**
1	nominal
Alternative in Minitab Stat Menu	
Stat/ Basic Statistics/ Paired t	
note …	
Y-Variable: normal distributed; Sample Size N: > 20	

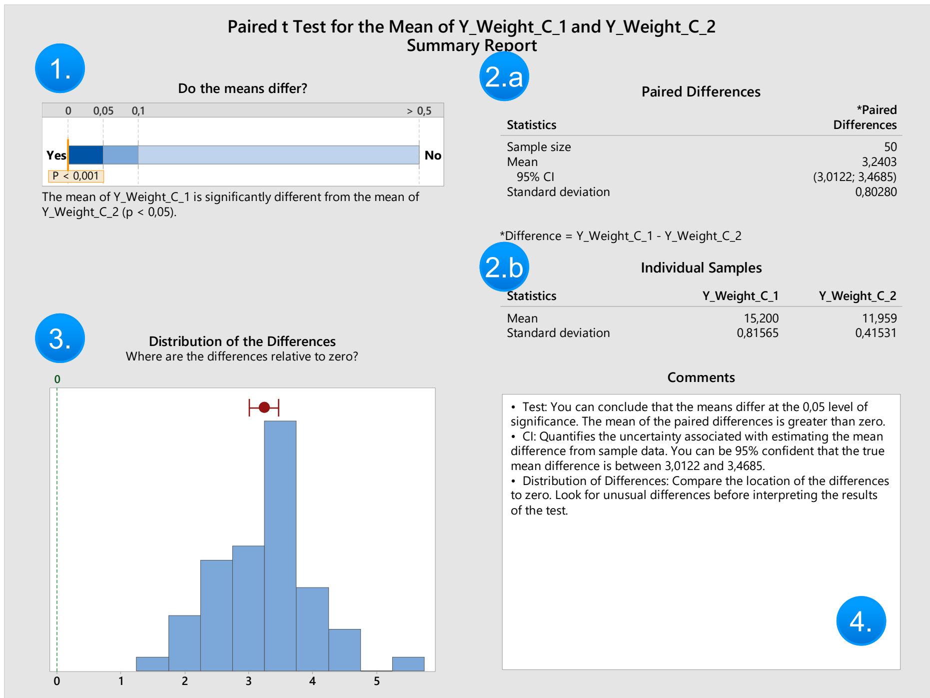


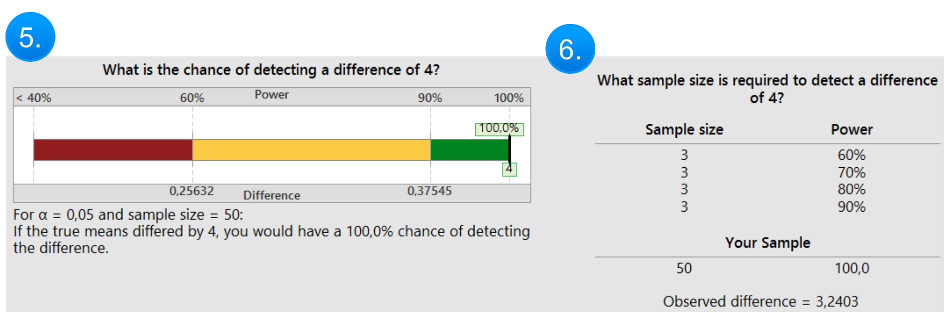
Dialog	
1. Sample Data	Two columns necessary for the Variables (no alternative grouping possible)
2. Type of Test	Mean of Sample Y_1 > Mean of Sample Y_2 (one-sided significance test)
	Mean of Sample Y_1 < Mean of Sample Y_2 (one-sided significance test)
	Mean of Sample Y_1 ≠ Mean of Sample Y_2 (two-sided significance test)
3. Alpha-Level	Significance level for the test
4. Power	Critical Difference between Samples, that need to be at least discriminable, if given

Example: Compare the Change in Weight of Cookies_raw vs. Cookies_baked

5.

Paired t-Test: Compare the Means of two dependent/ matched Variables





Results	
1. Significance Test	Do the Means differ?
	The bar of the chart indicates: - orange line: actual p-value of the Significance Test - dark blue sector: 0% > alpha <= 5% - light blue sector: 5% > alpha <= 10%
	accept H0 , if p> alpha, e.g. There are <u>no</u> differences accept HA, if p<= alpha, e.g. There is a difference
2. Statistics	a) Description of the Sample, Mean-Difference, Standard Deviation of Mean-Difference and 95% Confidence Interval (CI) for the Mean-Difference; b) Means, Standard Deviation of Means
3. Histogram	Distribution o the Difference between the Means
	Mean Difference and Confidence Intervals of the difference
	The Value: Difference= 0 (If the Confidence Interval includes the Difference 0 -> H0
5. Comments	Summary and comments about results
6. Power	Power (%) for critical Difference between Mean and Target, that needs at least to be identified
	The difference of >=4 can with the current sample size (N= 50) be identified with a probability of 100% (Rule of the Thumb: Power >= 80%)
7. Power	Power (%) for the to be identified critical difference between the Mean and the Target Value for different Sample Sizes (N)
	Example: the critical difference of 4 can be detected, if given, with a probability of 80%, if the Sample Size= 3 (I would not rely on this, due to the risk of sampling errors)

Example: Compare the Change in Weight of Cookies_raw vs. Cookies_baked

6.

2-Sample Standard Deviation Test: Compare the Standard Deviations of two Variables

Test

2-Sample Standard Deviation

Purpose

Compare the Standard Deviations of two independent variables (Y1 vs. Y2)

Hypothesis

Difference	There is a/ no Difference in: Standard Deviation between: Variables (Y_1 vs. Y_2)

Example

Compare the Standard Deviations of Cycle Time between Experts vs. Beginners

Comparison of the variation of Six Sigma Competence before vs. after Training

Comparison of the Distribution of Chocolate Pieces for 2 different stirring durations

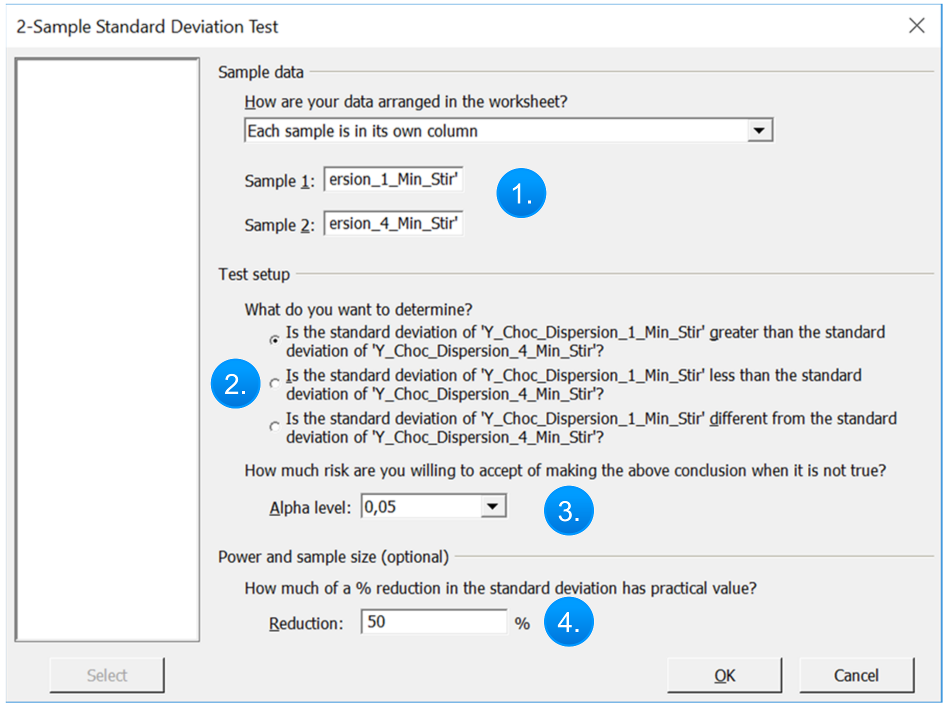
Y	Scale Level
1	cardinal
x	Scale Level
1	nominal

Alternative in Minitab Stat Menu

Stat/ Basic Statistics/ 2 Variances

note …

Y-Variable: normal distributed; Sample Size N: > 20

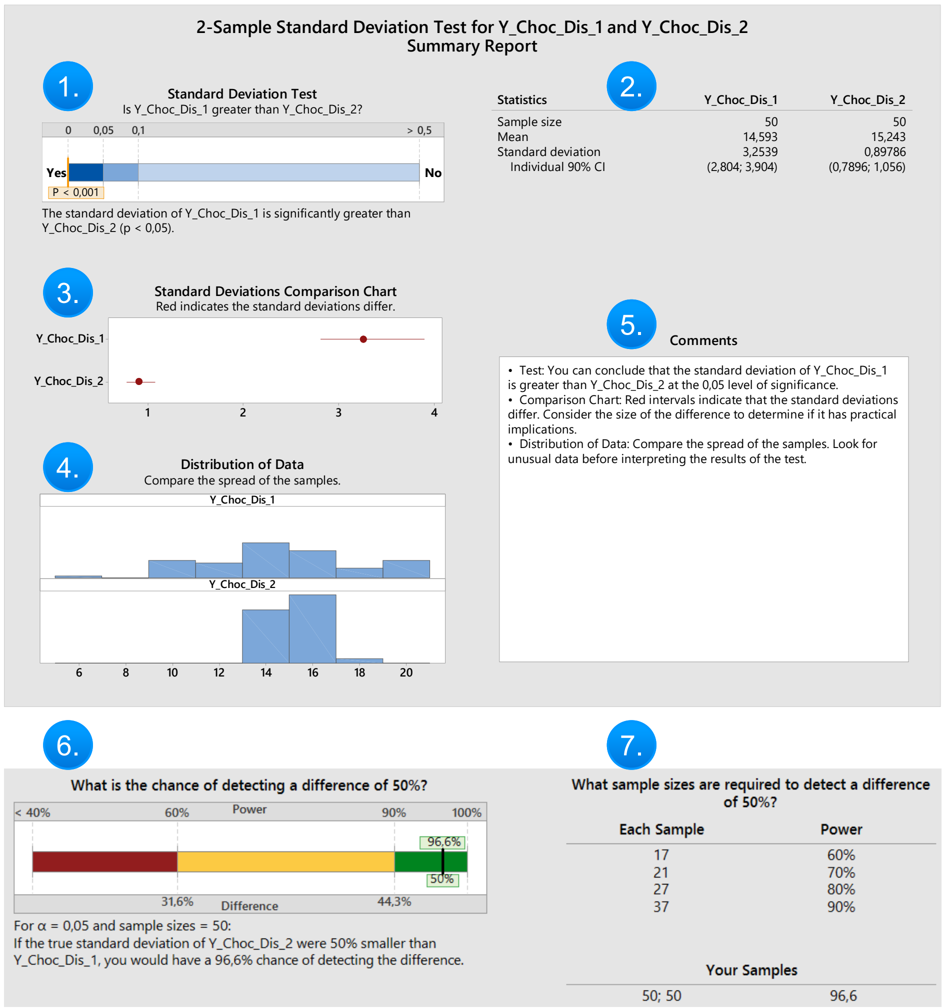


Dialog	
1. Sample Data	Arrangement of Data: see slide: Grouped data can be arranged in two alternative ways two cells with the measured data of both variables
2. Type of Test	Mean of Sample Y_1 > Mean of Sample Y_2 (one-sided significance test)
	St. Dev. of Sample Y_1 < St. Dev. of Sample Y_2 (one-sided significance test)
	St. Dev. of Sample Y_1 ≠ St. Dev. of Sample Y_2 (two-sided significance test)
3. Alpha-Level	Significance level for the test
4. Power	Critical Difference between Samples, that need to be at least discriminable, if given

Example: Compare the Distribution of Chocolate Pieces for 2 different stirring durations

6.

2-Sample Standard Deviation Test: Compare the Standard Deviations of two Variables



Results		
1. Significance Test	Do the Standard Deviations differ?	
	The bar of the chart indicates: - orange line: actual p-value of the Significance Test - dark blue sector: 0% > alpha <= 5% - light blue sector: 5% > alpha <= 10%	
	accept H0 , if p> alpha, e.g. There are no differences accept HA, if p<= alpha, e.g. There is a difference	
2. Statistics	Sample Size, Mean, Standard Deviation and 95% Confidence Interval (CI) for the Standard Deviation	
3. Interval Chart	Standard Deviation with Confidence Intervals for: Y1 vs.Y2 (Test: H0: Confidence Intervals intersect ; HA: CI do not intersect)	
4. Histogram	Distributions of the Values Y1 and Y2	
5. Comments	Summary and comments about results	
6. Power	Power (%) for critical Difference between Mean and Target, that needs at least to be identified	
	The difference of >=50% can with the current sample size (N= 50) be identified with a probability of 96,6% (Rule of the Thumb: Power >= 80%)	
7. Power	Power (%) for the to be identified critical difference between the Mean and the Target Value for different Sample Sizes (N)	
	Example: the critical difference of 50 can be detected, if given, with a probability of 80%, if the Sample Size= 27	

Example: Compare the Distribution of Chocolate Pieces for 2 different stirring durations

7.

ANalysisOfVAriance: Comparison of the Means of > 2 Variables

Test

One-Way ANOVA

Purpose

Compare the differences in the Means of a dependent variable (Y) in respect to factorial scaled independent variable (x)

Hypothesis

Difference There is a/ no Difference in: Mean of Y between: Factor Levels of x

Example

Compare the Cycle Time of more than 2 Processes

Vergleich der Bearbeitungszeiten des eines Prozesses an mehr als 2 Standorten
Vergleich des Ressourcenverbrauchs vor vs. nach Verbesserung

Compare the Taste of >2 Cookie-Types

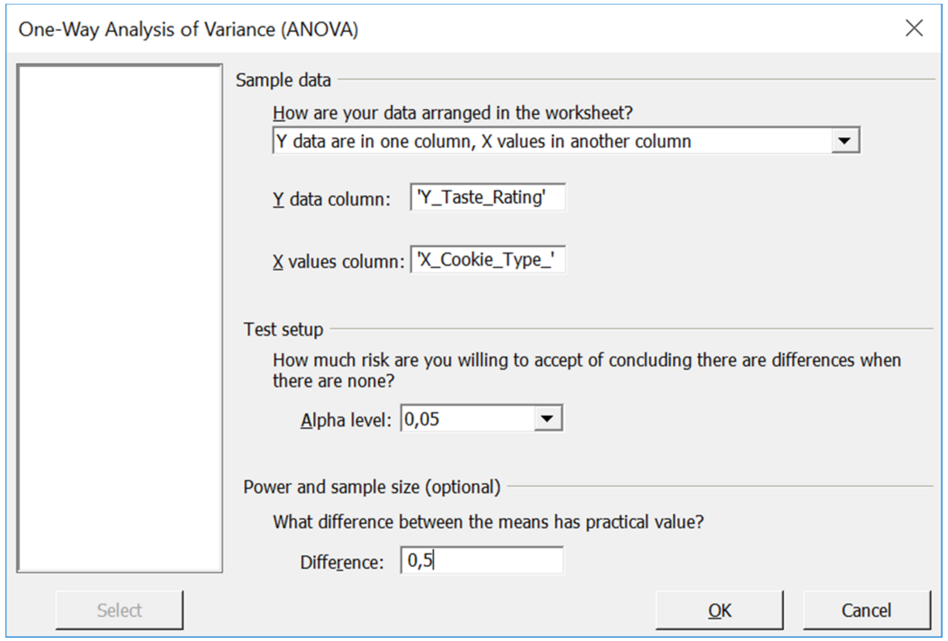
Y	**Scale Level**
1	cardinal
x	**Scale Level**
1	nominal / ordinal

Alternative in Minitab Stat Menu

Stat/ ANOVA/ One-Way or other

note …

Y-Variable: normal distributed; Sample Size N: > 20; N of all Factorial Levels must be the same

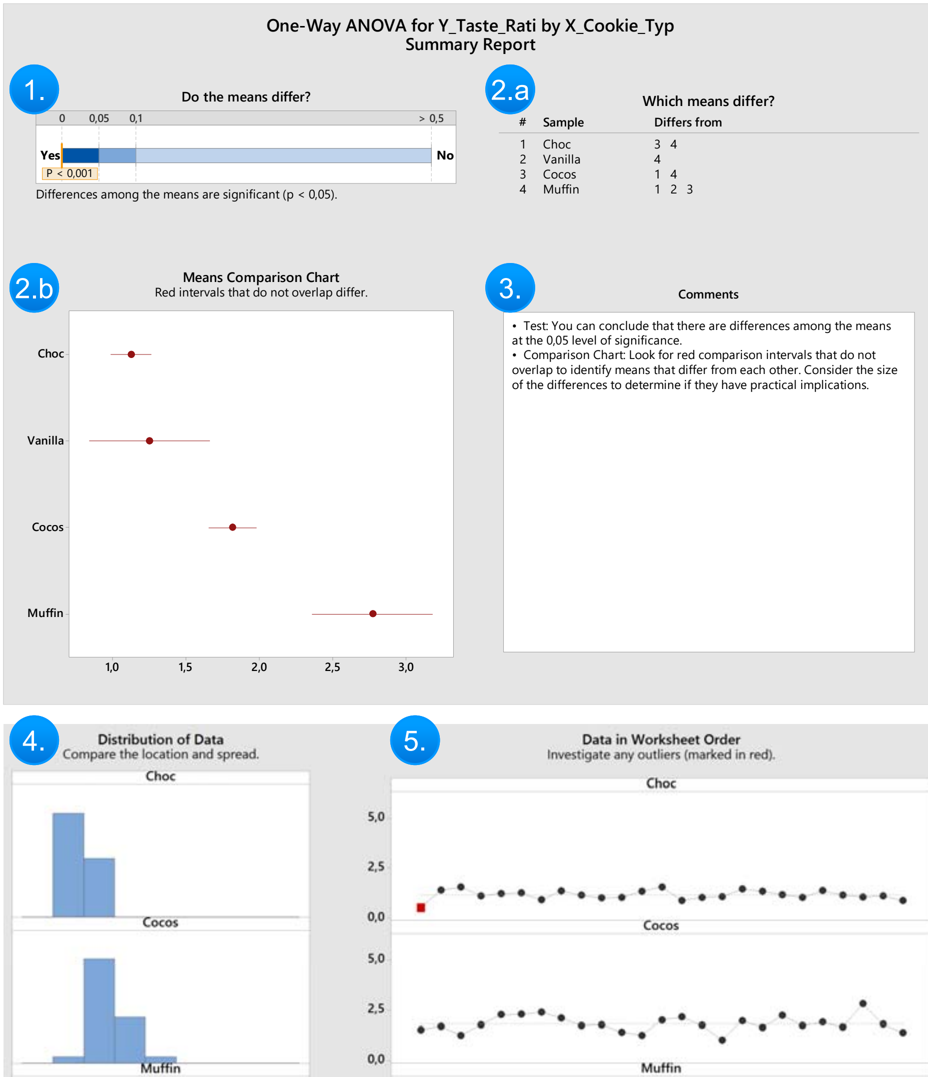


Dialog	
1. Sample Data	Arrangement of Data: see slide: Grouped data can be arranged in two alternative ways two cells with the measured data of both variables
2. Y-Column	Y (Cardinal Scaled Results)
2. X-Column	X (Nominal Scaled Factorial Levels)
3. Alpha-Level	Significance level for the test
4. Power	Critical Difference between Samples, that need to be at least discriminable, if given

Example: Compare the Taste of >2 Cookie-Types

7.

ANalysisOfVAriance: Comparison of the Means of > 2 Variables



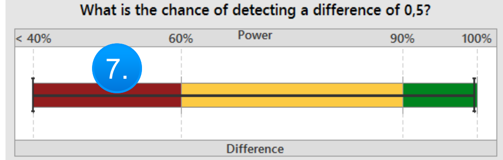
Results	
1. Significance Test	Do the Means differ?
	The bar of the chart indicates: - orange line: actual p-value of the Significance Test - dark blue sector: 0% > alpha <= 5% - light blue sector: 5% > alpha <= 10%
	accept H0 , if p> alpha, e.g. There are no differences accept HA, if p<= alpha, e.g. There is a difference
2. Comparison of Means	a) Table indicates sign. Differences between every xi and every other Factor Level of x b) Interval Plot indicates sign. Differences by red/ non overlapping CI´s for x´s
3. Comments	Summary and comments about results
4. Histogram	Distribution of Y (for the different Factor Levels of x)
5. Time Series Plot	Time Series Plot of Y with Outliers (for the different Factor Levels of x)
6. Statistics	N, Mean, Standard Deviation and CI for Mean
7. Power	Power (%) for critical Difference between Mean and Target, that needs at least to be identified
	The difference of >= 0,5 can with the current sample size (N= 25) be identified with a probability of 25,6 - 99,7% (Rule of the Thumb: Power >= 80%) (The Differences in the Intervals result from the variations in the Standard Deviations of the Factor Levels of x)
	Power (%) for the to be identified critical difference between the Mean and the Target Value for different Sample Sizes (N)
	Example: the critical difference of 0,5 can be detected, if given, with a probability of 80%, if the Sample Size= 94

6.

Statistics

X_Cookie_Typ	Sample Size	Mean	Standard Deviation	Individual 95% CI for Mean
Choc	25	1,1269	0,23734	(1,0289; 1,2249)
Cocos	25	1,8194	0,40873	(1,6507; 1,9881)
Muffin	25	2,7730	1,0259	(2,3496; 3,1965)
Vanilla	25	1,2538	1,0255	(0,83056; 1,6771)

What is the chance of detecting a difference of 0,5?



Based on your samples and α level (0,05), the chance of detecting a difference of 0,5 ranges from 25,59% to 99,68%.

7.

What sample sizes are required to detect a difference of 0,5?

All Samples	Power
62	60,1 - 100,0%
76	70,2 - 100,0%
94	80,3 - 100,0%
121	90,0 - 100,0%

Example: Compare the Taste of >2 Cookie-Types

8.

Chi-Square % Defective Test: Compare the percentual amounts Y of different factor levels of x

Test

Chi-Square % Defective

Purpose

Compare the percentual amounts Y of different factor levels of one attribute x (e.g. % defective vs. not; % sold vs. not)

Hypothesis

Difference	There is a/ no Difference in: % amounts of (Y) between: Factor Levels of Attribute (Xi)

Example

Compare the amount of Defects for different Processes

Compare the number of car accidents by car manufacturer

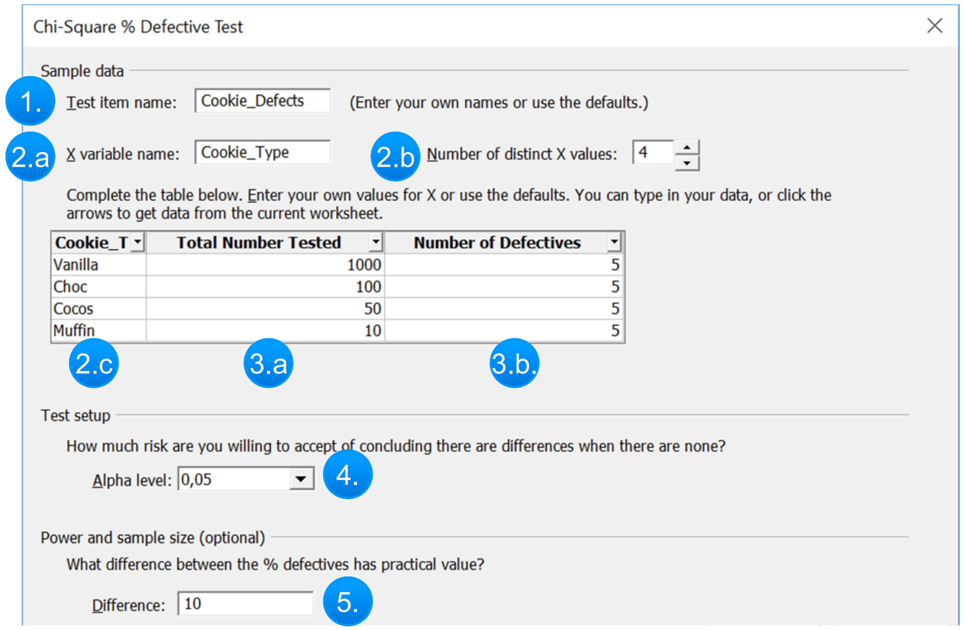
Compare the amount of Defects (Y) for different Cookie-Types (Xi)

Y	Scale Level
1	nominal (counted > discrete cardinal)
x	Scale Level
1	nominal

Alternative in Minitab Stat Menu

note …

Test does not have to be about defects, but about interesting portions in any other attribute X. - Number of Factor Levels of Xi can vary from: 3 - 12.



Dialog	
1. Test item name	Name of the Analysis
2.a X variable name	Name of x
2.b Number of X	Number of different factor levels/ categories of x (3 … 12)
2.c Factor Levels	Factor Levels/ Categories of X; Data can be imported from Worksheet or entered manually
3.a Total Number	Total Number of Units on each Factor Level/ Category x
3.b Number of Defects	Number of Units with interesting attribute (e.g. defects) on each Factor Level/ Category x
4. Alpha-Level	Significance level for the test
5. Power	Critical Difference between Samples, that need to be at least discriminable, if given

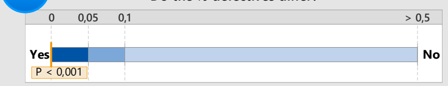
Example: Compare the percentual amounts of Defects for different Cookie-Types

8.

Chi-Square % Defective Test: Compare the percentual amounts Y of different factor levels of x

Chi-Square % Defective Test for Cookie_Defec by Cookie_Type
Summary Report

1. Do the % defectives differ?

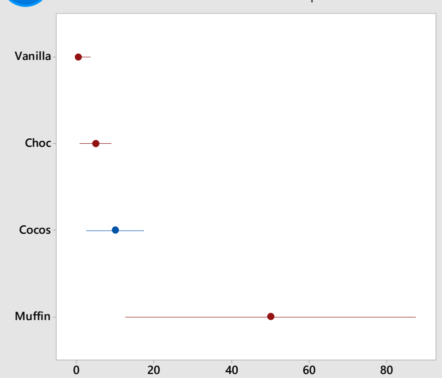


Differences among the % defectives are significant (p < 0,05).

2.a Which % defectives differ?

#	Cookie_Type	Differs from
1	Vanilla	4
2	Choc	4
3	Cocos	
4	Muffin	1 2

2.b % Defectives Comparison Chart
Red intervals that do not overlap differ.



3. Comments

- Test: You can conclude that there are differences among the % defectives at the 0,05 level of significance.
- Comparison Chart: Look for red comparison intervals that do not overlap to identify % defectives that differ from each other. Consider the size of the differences to determine if they have practical implications.

Results	
1. Significance Test	Do the % defectives differ?
	The bar of the chart indicates: - orange line: actual p-value of the Significance Test - dark blue sector: 0% > alpha <= 5% - light blue sector: 5% > alpha <= 10%
	accept H0 , if p> alpha, e.g. There are no differences accept HA, if p<= alpha, e.g. There is a difference
2. Comparison of %	a) Table indicates sign. Differences between every xi and every other Factor Level of x b) Interval Plot indicates sign. Differences by red/ non overlapping CI´s for x´s
3. Comments	Summary and comments about results
4. Statistics	oberseved and expected frequencies, (from the deviations the Chi^2 statistic is calculated)
5. Statistics	Number of tested Units, defective Units, % defect Units and Confidendence Interval (CI) for % Defectives
6. Power	Power (%) for critical Difference between Mean and Target, that needs at least to be identified
	The difference of >= 10% can with the current varying sample sizes (10 .. 1000) be identified with a probability of 97,7 - 100% (Rule of the Thumb: Power >= 80%)
	Power (%) for the to be identified critical difference between the Mean and the Target Value for different Sample Sizes (N)
	Example: the critical difference of 10% can be detected, if given, with a probability of 80%, if the Sample Size>= 26 in each Category/ Factor Level

4. Number of Defective and Nondefective Items

Cookie_Type	Defective Observed	Defective Expected	Nondefective Observed	Nondefective Expected
Vanilla	5	17,2	995	983
Choc	5	1,72	95	98,3
Cocos	5	0,862*	45	49,1
Muffin	5	0,172*	5	9,83

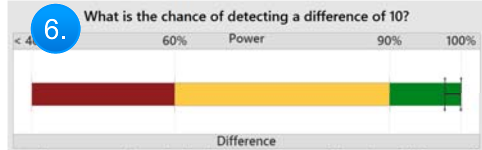
* Indicates a violation.

- To ensure validity of the test, the expected number of defectives and nondefectives should be at least 1,5.
- To ensure validity of the comparison intervals, the observed number of defectives and nondefectives should be at least 5.

5. Statistics

Cookie_Type	Number Tested	Defectives	% Defective	Individual 95% CI
Vanilla	1000	5	0,50	(0,16; 1,16)
Choc	100	5	5,00	(1,64; 11,28)
Cocos	50	5	10,00	(3,33; 21,81)
Muffin	10	5	50,00	(18,71; 81,29)

6. What is the chance of detecting a difference of 10?



Based on your samples and α level (0,05), the chance of detecting a difference of 10 ranges from 97,74% to 100,00%.

What sample sizes are required to detect a difference of 10?

All Samples	Power
17	60%
21	70%
26	80%
34	90%

Example: Compare the percentual amounts of Defects for different Cookie-Types

9.

Chi-2 Test for Association: Compare the frequencies (Y) of 2 concatenated Variables (Xi, Xj)

Test

Chi-Square Test for Association

Purpose

Compare the percentual amounts of Y in respect to the factorial levels of 2 categorial Variables (Xi, Xj)

Hypothesis

Difference	There is a/ no Difference in: Frequencies of Yij between: the conditions of Xi/ Xj

Example

Compare the Salary (Y) in respect to Profession (Xi) and Country (Xj)

Compare the number of car accidents (Y) by car manufacturer (Xi) and Level of Expertise of Driver (Xj)

Compare the amount of sold Cookies (Y), differentiated by Type (Xi) and Continent (Xj)

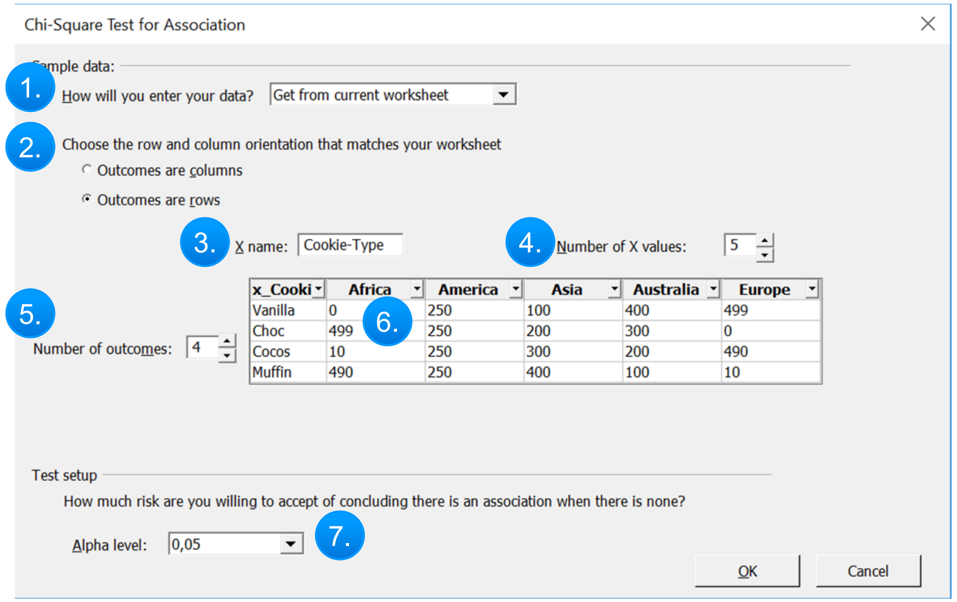
Y	Scale Level
1	nominal (counted > discrete cardinal)
x	Scale Level
2	nominal

Alternative in Minitab Stat Menu

Stat/ Tables/ Chi-Square-Test for Association

note ...

Number of Factor Levels for Xi and Xj can vary from: 3 - 6.

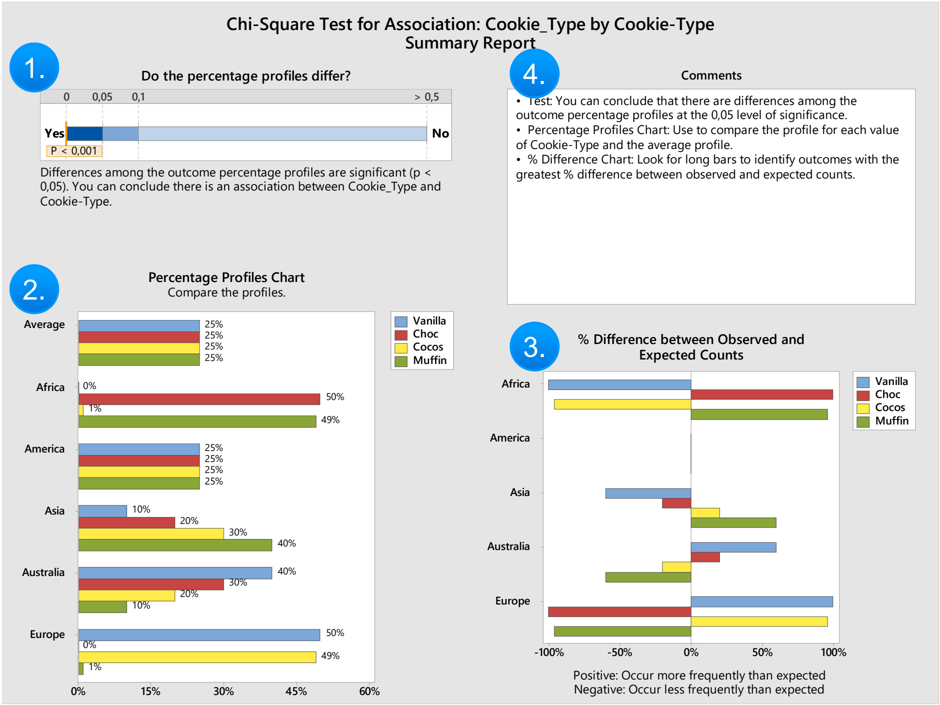


Dialog	
1. Sample Data	The data can be a) imported from the Worksheet or b) entered directly into the data table
2. Orientation of the table	The table can be inverted. This switching of variables in the table influences their orientation in the result charts, but does not affect the resulks/ significance test
3. Name	descriptive name for Y/ X
4. Number of Categories	3-6 different categories are possible
4. Number of Categories	3-6 different categories are possible
6. Data Table	Data imported from the Worksheet through Drop-Down-Lists or entered directly
7. Alpha-Level	Significance level for the test

Example: Compare the amount of sold Cookies (Y), differentiated by Type (X1) and Continent (X2)

9.

Chi-2 Test for Association: Compare the frequencies (Y) of 2 concatenated Variables (Xi, Xj)



Results	
1. Significance Test	Do the percentage profiles differ?
	The bar of the chart indicates: - orange line: actual p-value of the Significance Test - dark blue sector: 0% > alpha <= 5% - light blue sector: 5% > alpha <= 10%
	accept H0 , if p> alpha, e.g. There are no differences accept HA, if p<= alpha, e.g. There is a difference
2. Percentage-Profile Chart	Percentual distribution of Y in Xi/ Xj pairings. The average describes the profile of variable over all Xi categories. The deviations of Xi on the different Levels of Xj shows the specific Xi/ Xj profile. The deviations are explicitly depicted in the % Difference chart.
3. % Difference Chart	The difference chart shows the differences in Xi and Xj between expected and observed values. The larger the difference, positive or negative, the greater the contribution to the Chi^2-Test-Value and thus to the significance of the results.
4. Comments	Summary and comments about results
5. Statistics	Number of observed vs. expected values for each Xi/Xj combination, as well as violations of conditions of the Chi2-Test

5.

Observed and Expected Counts

	Africa Obs	Africa Exp	America Obs	America Exp	Asia Obs	Asia Exp	Australia Obs	Australia Exp	Europe Obs	Europe Exp
Vanilla	0	250	250	250	100	250	400	250	499	250
Choc	499	250	250	250	200	250	300	250	0	250
Cocos	10	250	250	250	300	250	200	250	490	250
Muffin	490	250	250	250	400	250	100	250	10	250
Total	999		1000		1000		1000		999	

Expected counts should be at least 1 to ensure the validity of the p-value for the test.

Example: Compare the amount of sold Cookies (Y), differentiated by Type (X1) and Continent (X2)

The General Linear Model (GLM) allows the forecasting of Y, if x is known

Many parametric Statistical Tests are based on the General Linear Model:

$$y = b + ax + e$$

Dependent Variable → y

Error → e

Constant → b

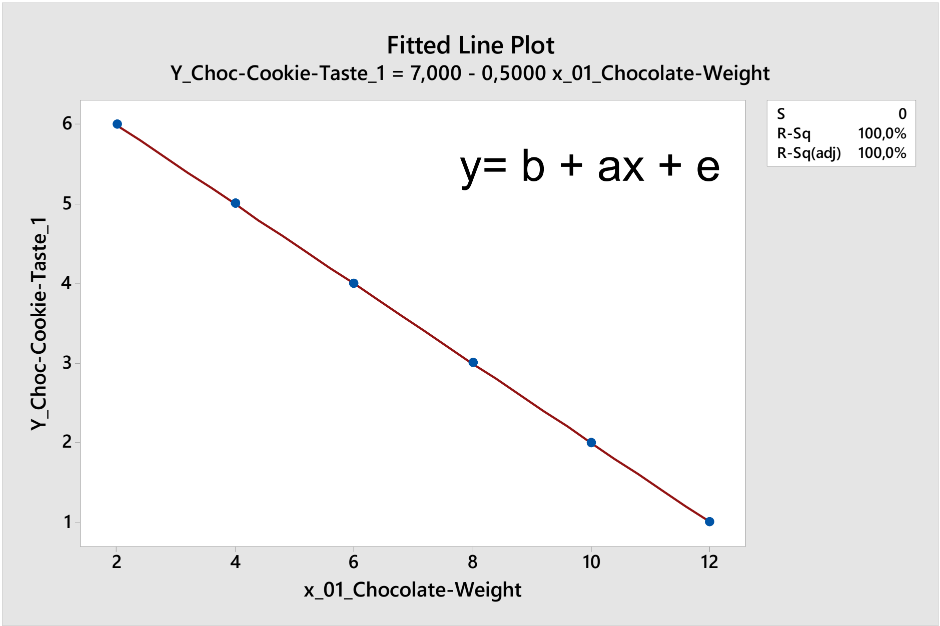
Independent Variable → x

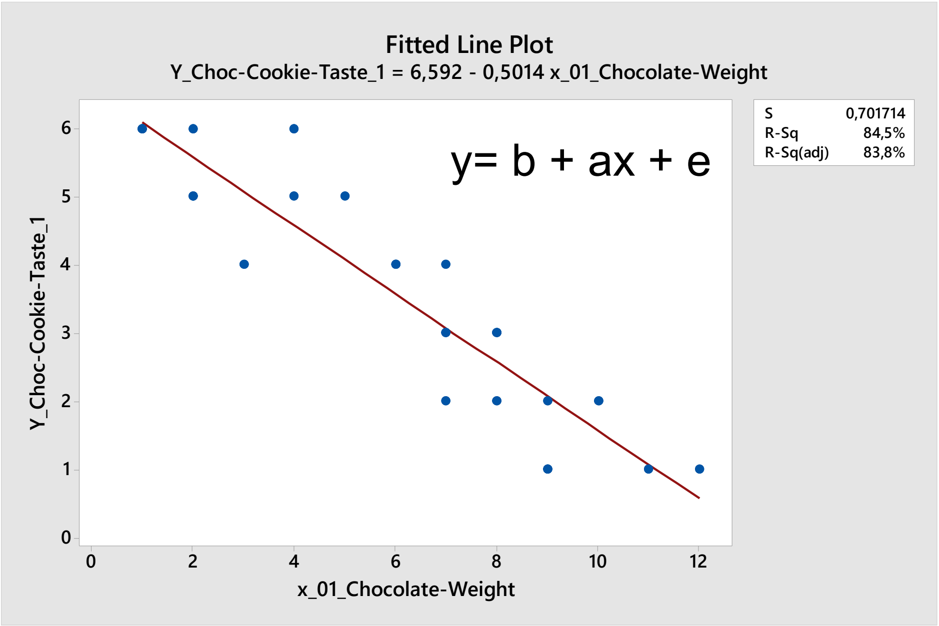
Weight of the independent variable (Effect) → a

- This regression line quantifies the relationship between **x** and **Y** (Effect).
- The higher **a**, the stronger the relationship. Since **a** is not a standardized value, the strength of the relationship between **x** and **Y** is expressed as a correlation.
- The determination coefficient R^2 (= squared correlation coefficient) expresses in percent how much the variability in **Y** can be explained by the variability in **x** (= explained portion of variability)
- The higher the correlation/ the determination coefficient, the smaller the error (e).
- By a perfect correlation (r=-1.00 ∨ r=1.00) respectively total determination (R^2= 1.00) is e= 0.

Examples for GLM based Tests are: Correlation, Regression, ANOVA, Factor-Analysis, Discriminant Analysis

The predictions from the general linear model (GLM) are more accurate, …





Perfect, positive relationship between:

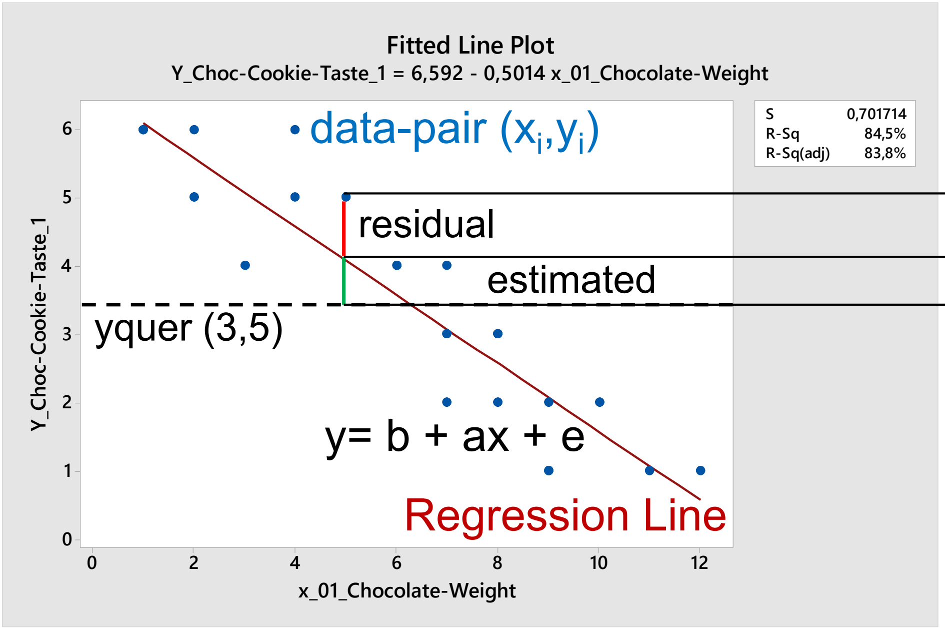
- X= Chocolate Weight
- Y= Taste
- b= 7 (y-intercept/ constant term)
- a= -0,5 (slope of regression line)
- r= 1,00 (correlation-coefficient)
- R^2= 100% (determination-coefficient)
- e= 0 (error-term)

strong, positive relationship between:

- X= Chocolate Weight
- Y= Taste
- b= 6,592 (y-intercept/ constant term)
- a= -0,5014 (slope of regression line)
- r= 0,837 (correlation-coefficient)
- R^2= 70,17% (determination-coefficient)
- e> 0 (error-term)

… the smaller the variability in the collection of values around the regression line (=Residuals).

Even though the relationship between Y and X is not perfect, …



$y_i - \hat{y}_i$

$\hat{y}_i - \bar{y}$ $\Big\}$ $y_i - \bar{y}$

Sum of Squares due to Regression $= \Sigma\, (\hat{y}_i - \bar{y})^2$ (SSR)

Sum of Squares due to Error $= \Sigma\, (y_i - \hat{y}_i)^2$ (SSE)

Sum of Squares total $= \Sigma\, (y_i - \bar{y})^2$ (SST)

SST = SSR + SSE

R^2= SSR/ SST

- The Regression Line is placed between the datapoints

  so that the squared deviations of all points to the line is minimized.
- Since the relationship is not perfect in work environments, there are always (residual-) deviations= errors (e).
- The stronger the a-weights, the steeper the slope up to 45°, the better Y can be estimated by x.
- Influences x with significant, i.e. from zero different **a**-weight is a suitable anchor point for improvements.

… the Determinmation with R2= 84% offers a good basis for forecasts and improvements

Correlation and Regression …

- Starting point for the Correlation/ Regression is the Scatter Plot.
- The X-axsis rshows **one** of the independent variables, i.e. the Influences in a Process (X_i, X_p)
- The Y-Achse shows the dependent variable, i.e. the attribute of an Output (Y)
- Hypothesis: There is a/ no (the … the …)Relatonship between x and Y (Y= f(x))

- The positions of the xY datapoints give a first impression of the degree of Relationship between x and Y.

- The Correlation-Coefficient: $\mathbf{r_{xY}}$ numerically indicates the degree of Relationship between x and Y.
- $\mathbf{r_{xY}}$ can vary between -1 and +1,

  \- 1 := perfect negative linear Relationship,

  0 := no Relationsdhip

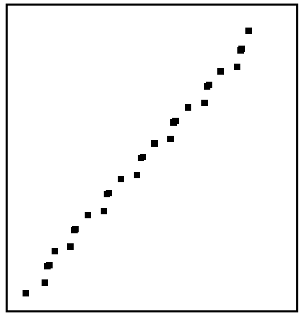
  \+ 1 := perfect positive linear relationship.

$$-1 \le r_{xy} \le 1$$

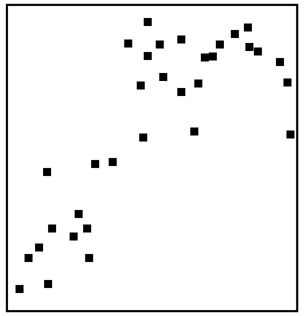
- The Regression Analysis calculates the influences of **multiple** independent variables (Xi, Xp) on the dependent variable (Y) at the same time.

Value of "r"	Definition
0,7 ≤ **r** ≤ 1	Strong positive correlation
0,3 ≤ **r** ≤ 0,7	Medium positive correlation
- 0,3 ≤ **r** ≤ +0,3	no correlation
- 0,7 ≤ **r** ≤ - 0,3	medium negative correlation
- 1 ≤ **r** ≤ - 0,7	strong negative correlation

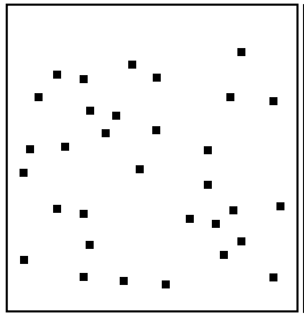
r = 0,85



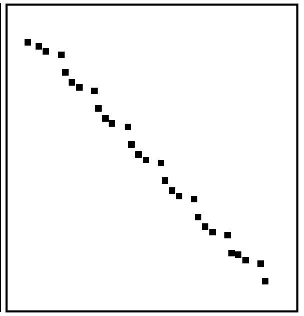
r = 0,5



r = 0,1



r = - 0,8



… are statistical hypothesis tests

10.

Regression: Y= f(x) - Relationship between cardinally scaled variables x and Y

Test

Regression

Purpose

Analyse the relationship between (multiple) x and Y

Hypothesis

Relationship	There is a/ no Relationship between: Influence (x) and: Result (Y)

Example

Relationship between complexity of task (x) and Cycle Time (Y)

Relationship between speed (x) and fuel consumption (Y)

Relationship between baking time (x) and browning degree of the Cookie (Y)

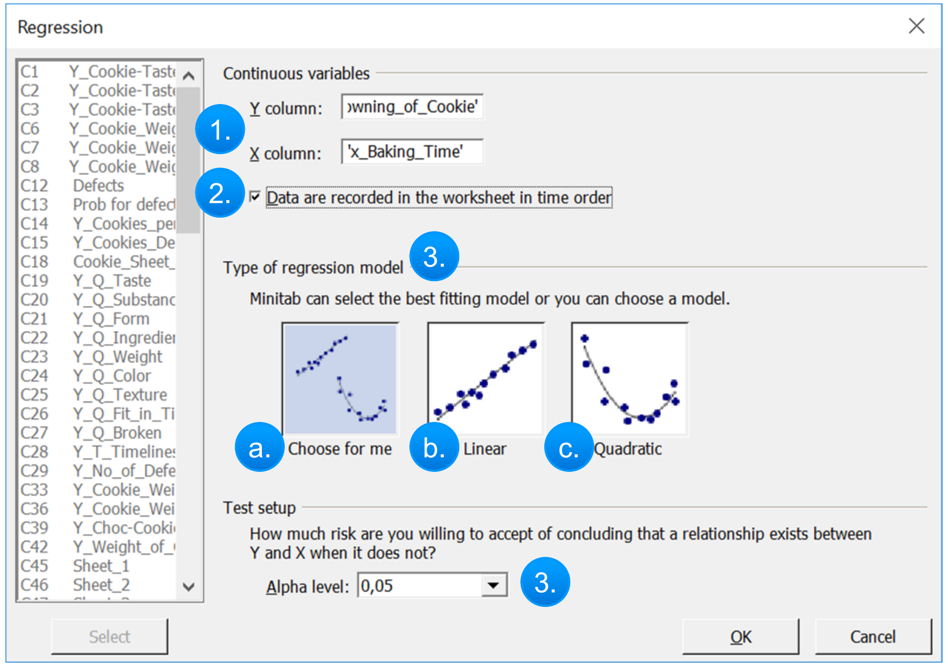
Y	**Scale Level**
1	cardinal
x	**Scale Level**
n	cardinal

Alternative in Minitab Stat Menu

Stat/ Regression

note …

X- and Y-Variables: normal distributed; Sample Size N: > 15



Dialog	
1. Sample Data	Y := cardinal scaled variable of Results x := cardinal scaled variable of Influence
2. Order of Data	If data are collected in time order then the time dependency of the Residuals can be shown, if present
3. Type of Model	Based on the type of assumed Relationship between x and Y the type of model can be selected, that will be tested
a)	Choose for me: Minitab chooses the model with the best adjustments according to the collected data
b)	Linear: Influence is represented as x in the Model to predict Y
c)	Quadratic: Influence is represented as x and x2 in the Model to predict Y
	Linear Model is to be preferred if the Determination Coefficients (R2) of the tested Models are of similar size
4. Alpha-Level	Significance level for the test

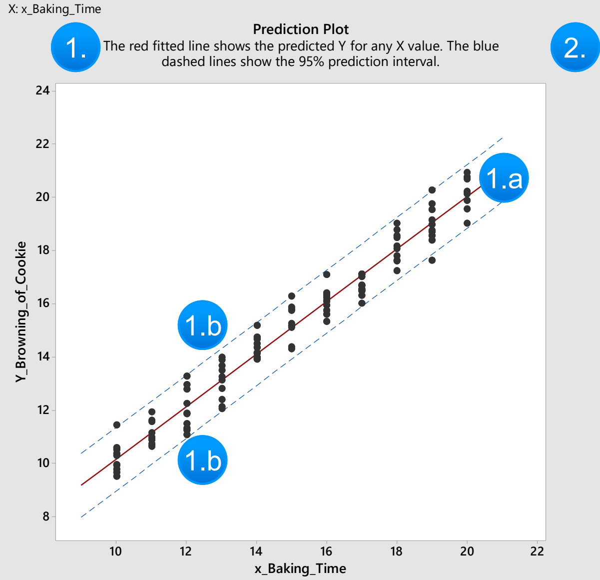
Example: Relationship between baking time (x) and browning degree of Cookie (Y)

10.

Regression: Y= f(x) - Relationship between cardinally scaled variables x and Y

Regression for Y_Browning_of_Cookie vs x_Baking_Time
Prediction Report

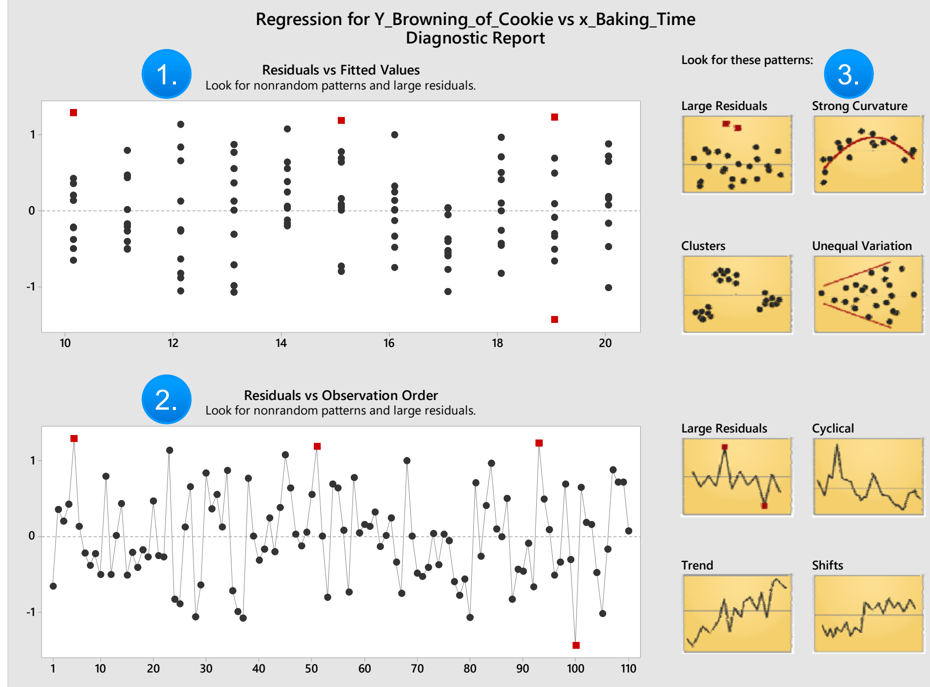
Y: Y_Browning_of_Cookie
X: x_Baking_Time



2.

X	Predicted Y	95% PI
9	9,1653	(7,9627; 10,368)
9,5	9,6599	(8,4602; 10,860)
10	10,154	(8,9576; 11,351)
10,5	10,649	(9,4546; 11,843)
11	11,144	(9,9514; 12,336)
11,5	11,638	(10,448; 12,828)
12	12,133	(10,944; 13,321)
12,5	12,627	(11,440; 13,814)
13	13,122	(11,936; 14,308)
13,5	13,616	(12,432; 14,801)
14	14,111	(12,927; 15,295)
14,5	14,606	(13,422; 15,789)
15	15,100	(13,916; 16,284)
15,5	15,595	(14,411; 16,778)
16	16,089	(14,905; 17,273)
16,5	16,584	(15,399; 17,769)
17	17,078	(15,893; 18,264)
17,5	17,573	(16,386; 18,760)
18	18,067	(16,879; 19,256)
18,5	18,562	(17,372; 19,752)
19	19,057	(17,864; 20,249)
19,5	19,551	(18,357; 20,746)
20	20,046	(18,849; 21,243)
20,5	20,540	(19,341; 21,740)
21	21,035	(19,832; 22,238)

To obtain additional predicted values, right-click the graph and use the crosshairs tool.



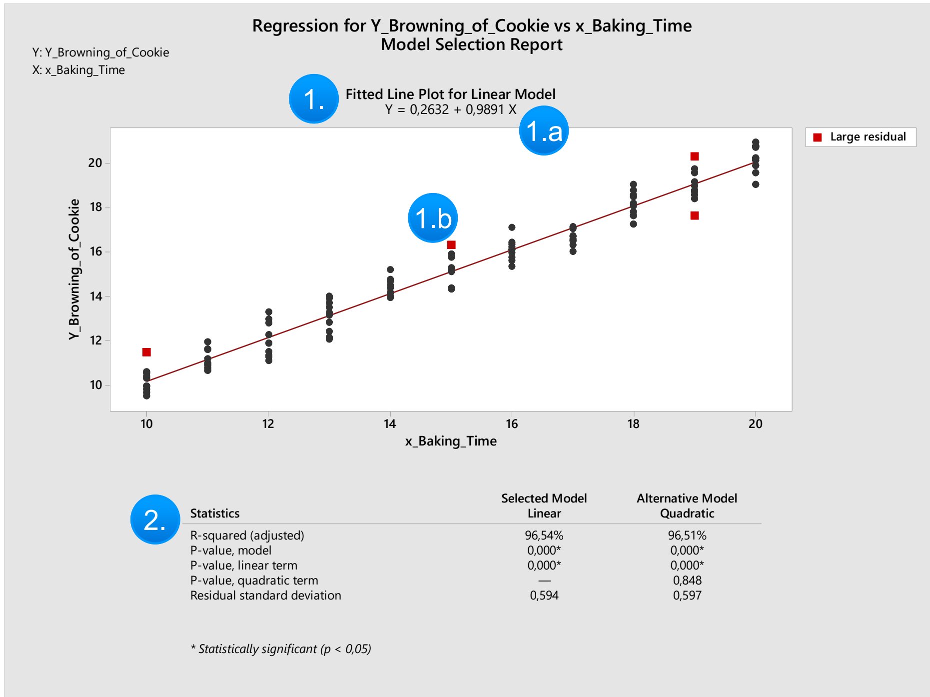
Results	
1. Prediction Plot	Scatter Plot for the Variables x and Y
a)	Regression Line
b)	95% Prediction Intervall (PI), i.e. the interval in which the predicted value of Y will be with a 95% confidence, for a given x
2. Statistics	Values for x, the predicted Y and ist 95% Prediction Interval (PI)

Results	
1. Residuals vs. Fitted	Plot shows the Residuals, i.e. the deviation of the data points from the Regression Line, along the scale of x, i.e. from small to large values of x
1. Residuals vs. Observation Order	Plot shows the Residuals, i.e. the deviation of the data points from the Regression Line, along the time order of the collected data, as given in the Worksheet, i.e. from first to last collected data of x
3. Signals	Signals as different patterns which show, that the Residuals are systematically influenced; try to identfy the Root-Causes of these patterns and eliminate them;

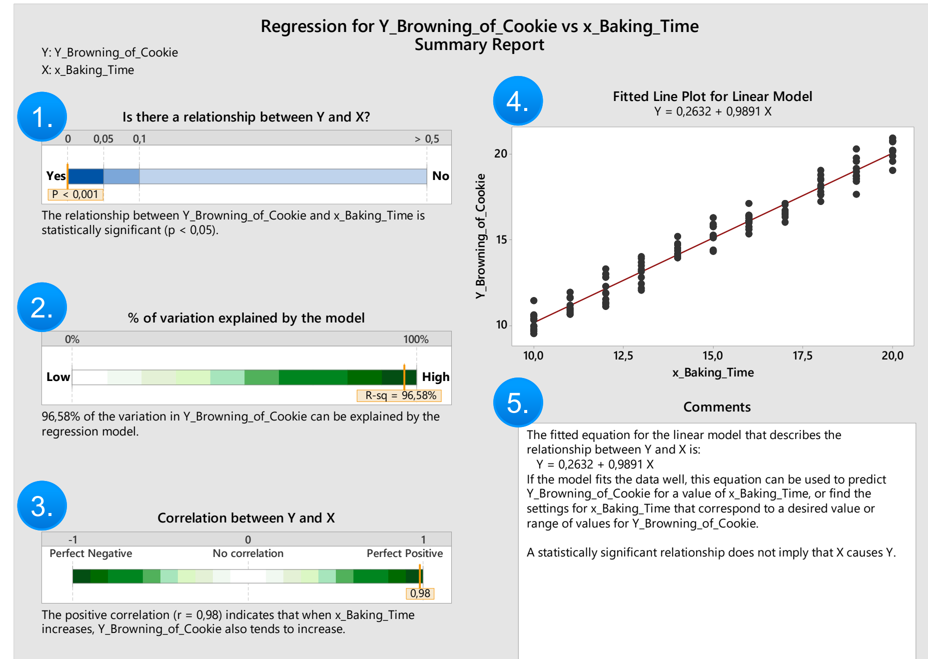
Example: Relationship between baking time (x) and browning degree of Cookie (Y)

10.

Regression: Y= f(x) - Relationship between cardinally scaled variables x and Y



Results	
1. Fitted Line Plot	Scatter Plot for the Variables x and Y
a)	Regression Equation for the Prediction of Y by the values of x
b)	Large Residuals (if the Root-Cause of these deviating values is known, then they might be exluded from the calculation)
2. Statistics for the tested Models	- R2 (adjusted): Percentual degree of variation of Y explained by x - p-values for the tested Models

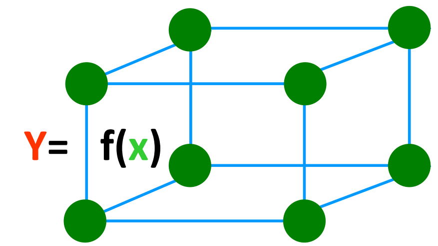


Results	
1. Significance Test	Is there a Relationship between X and Y? The bar of the chart indicates: - orange line: actual p-value of the Significance Test - dark blue sector: 0% > alpha <= 5% - light blue sector: 5% > alpha <= 10% accept H0 , if p> alpha, e.g. There are no differences accept HA, if p<= alpha, e.g. There is a difference
2. % variation explained	The determination coefficient (R-squared/ R2) shows the percentage of variation of Y that can be explained by the variation of x. The remaining percentage is the error portion.
3. Correlation	Level of Correlation between x and Y, which can be positive or negative. (The correlation rxy in this univariate Model (only one X) is the squareroot of R2.)
4. Scatter Plot	Scatter Plot for X and Y with Regression Line and the Regression Equation
5. Comments	Summary and comments about results

Example: Relationship between baking time (x) and browning degree of Cookie (Y)

Design of Experiments (DoE)

The DoE is a systematic approach …



Design of Experiments is:

- an effective and efficient method to analyse
- Cause- & Effect Relationships between
- Influences of the Input and Process (X_I, X_M, X_R) and
- Attributes of the Output, like Quality, Availability and Resource Consumption (Y)

The DoE:

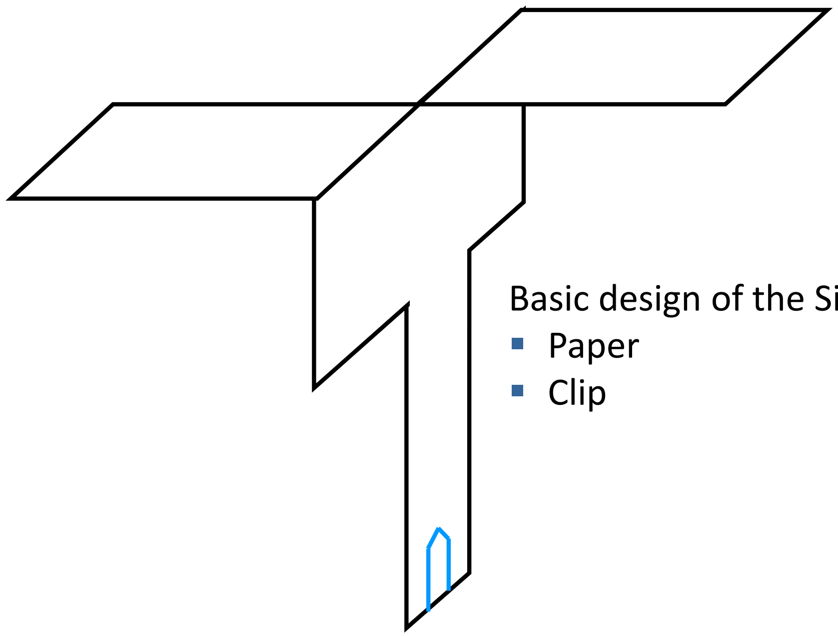
- identifies Variables (X) with the **highest impact** (Main Effects) on the result of the Outputs (Y) and thus can be seen as Root-Causes of the Variation
- identifies **interdependencies** (Interactions) between different levels of at least two Influences (X) on the result of the Outputs (Y)
- **quantifies** and thus **predicts**, how and to which degree the variation of Influences (X) affect the result of the Outputs (Y)
- identifies the specific **adjustments** for the Influences (X) to optimize the results of the Outputs (Y) in direction of a target

… to identify the best Solutions

SigmaCopter AG

Situation

- You are a Engineer in the Development Department of the SigmaCopter AG.
- Your Customer require longer flight durations of the SigmaCopter.
- Thus your Company decides to improve the actual aircraft model.



Basic design of the SigmaCopter
- Paper
- Clip

What is your approach?

SigmaCopter: Task (1/2)

Target of the Project:

- Optimize the duration of the free fall from the ceiling of this room (x meter) by:
- Development and Test of Prototypes

Legal conditions:

- It is not allowed to change the basic design of the SigmaCopter (e.g. „paper planes“ are not admitted)
- Budget for material and tests is limited to 2.500.000 €.
- Each Prototype can be tested in repeated measurements but:
  It is not allowed to modify a configured and tested Prototypes to serve as another Prototype

Allowed Tools:

- scissors, glue stick, timer

- **Zeit:** 90 min.

				Rotor			
				short		long	
				Fuselage		Fuselage	
				straight	cut	straight	cut
Size	big	Clip	yes				
			no				
	small	Clip	yes				
			no				

Example

SigmaCopter: Task (2/2)

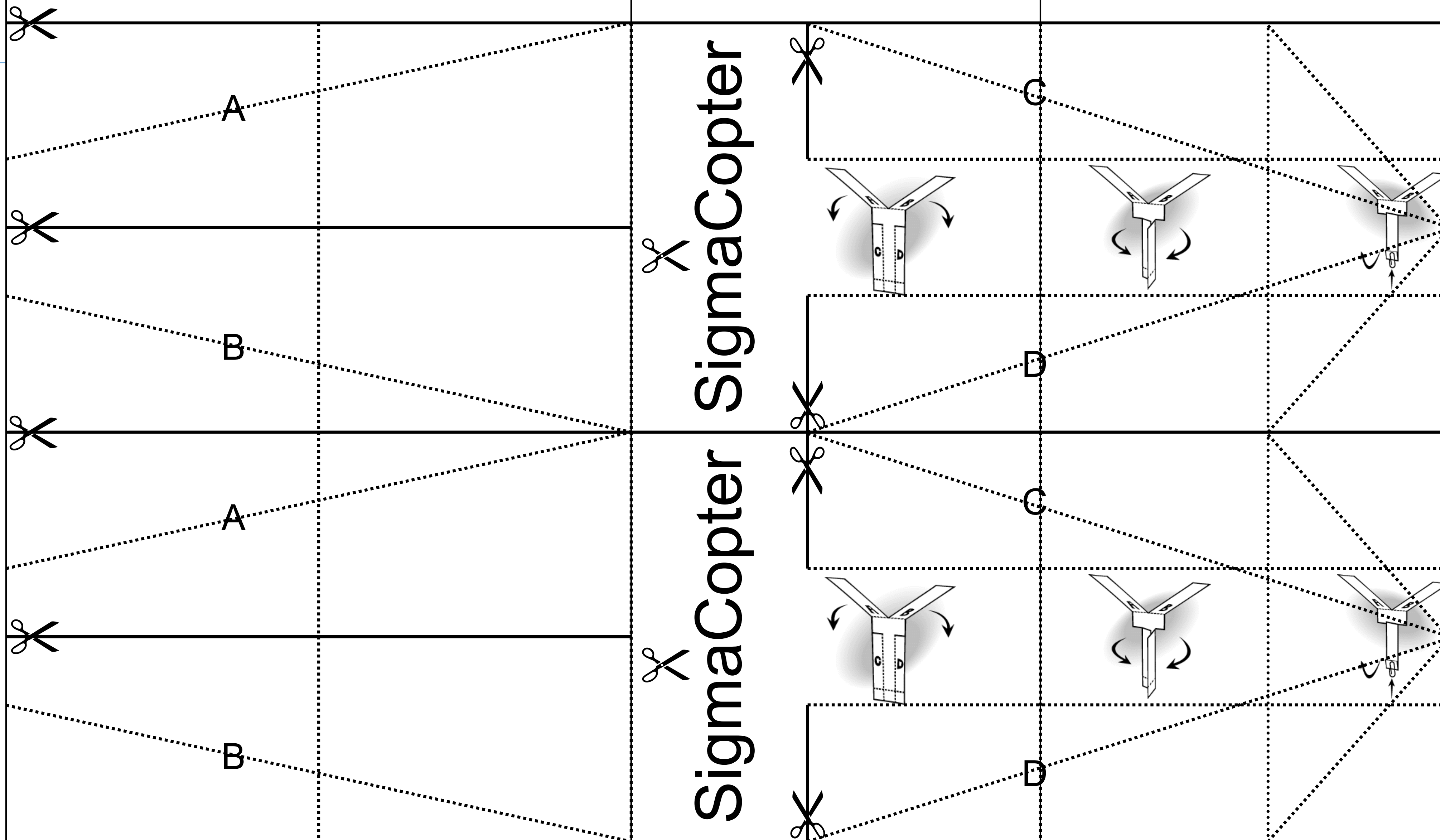
Use a sequential experimentation process**

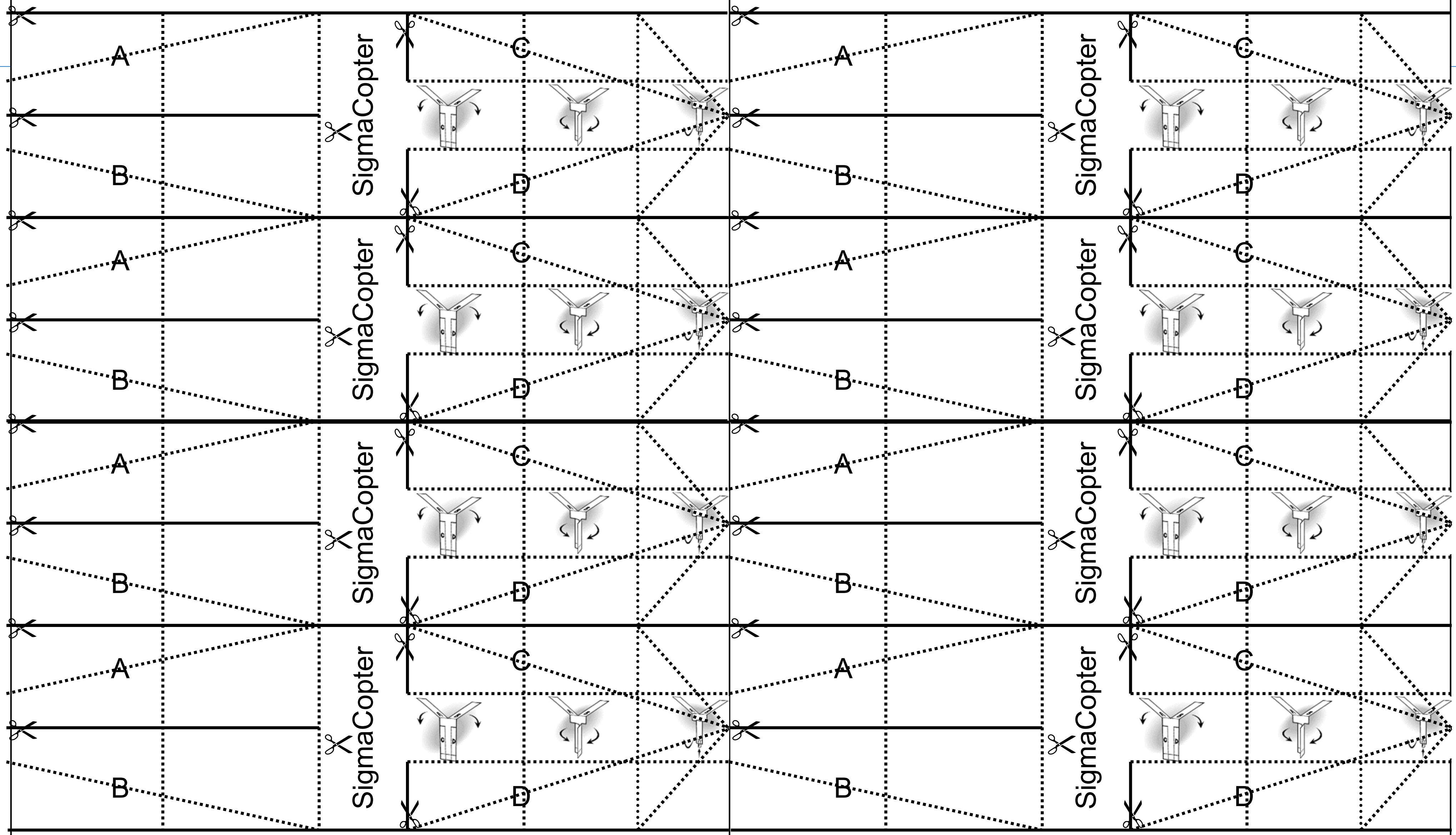
- A sequential experimentation approach uses a sequence of smaller experiments where the results at each stage guide the experimentation at the next stage. An advantage of the sequential approach is that at each stage, only a small number of experimental trials are run so that you are less likely to waste resources.
- A typical sequential experimentation process includes several stages. You should only use the stages that are appropriate for your situation. Typically, the stages include the following:
  - **Preliminary screening**: Create a list of potential factors and then eliminate unimportant factors using brainstorming, hypothesis tests, graphical analysis, or other tools.
  - **DOE Screening**: Use a screening experiment when you need to reduce the number of factors further. In the Assistant, screening experiments examine the main effects of 6–15 factors to help you identify the critical few factors that influence the response.
  - **Modeling**: Use a modeling design to construct a model that describes the relationship between the response and the critical factors. In the Assistant, a modeling design examines main effects and interactions for 2–5 factors and looks for curvature in the continuous factors. If curvature is detected, the Assistant will add experimental trials that will allow you to fit a quadratic model.
  - **Optimization**: Use the final model to search for an optimal solution. In the Assistant, you can identify optimal settings for each factor, if that is important for your process.

Your available time: 90 min.

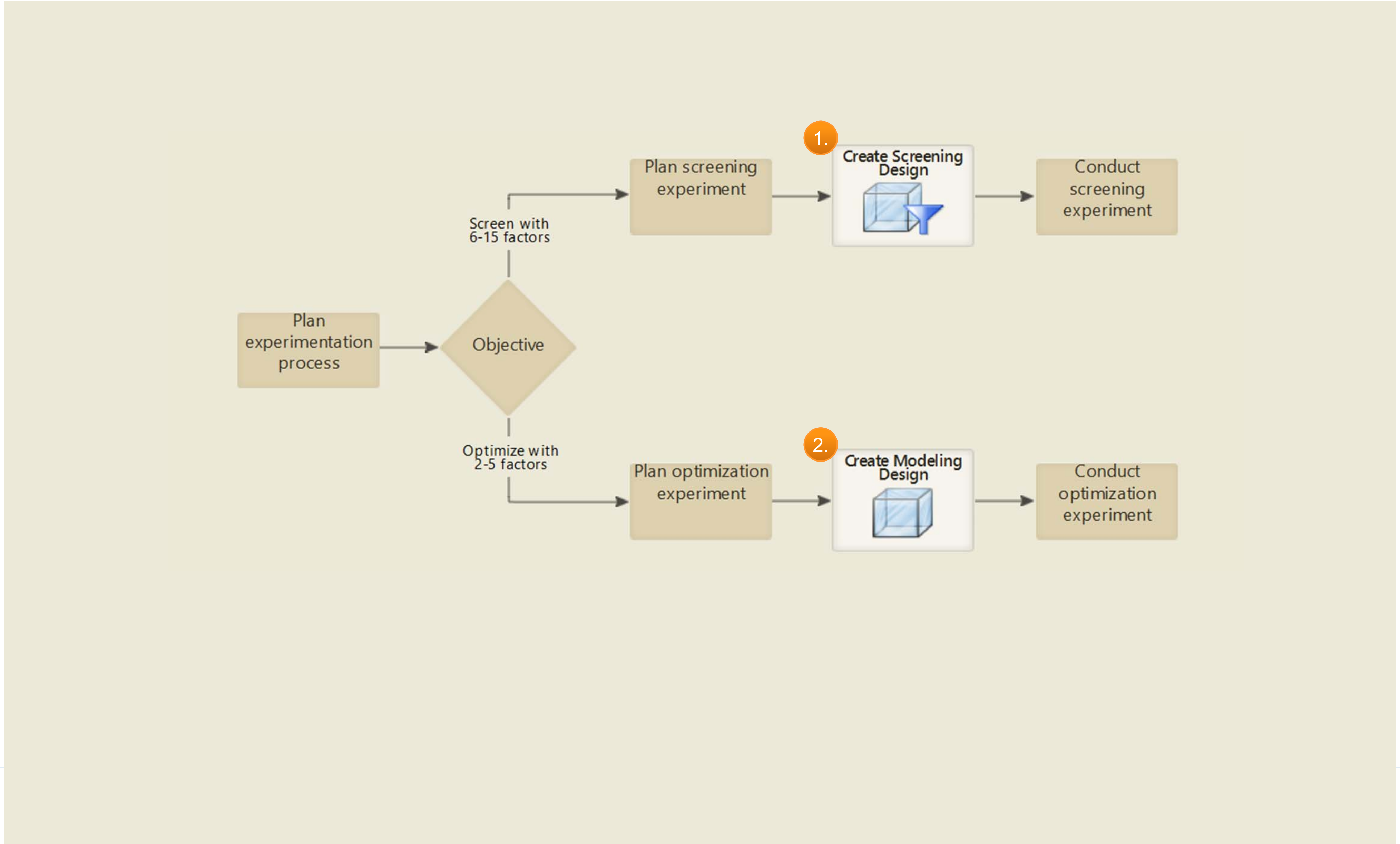
Tips: https://www.minitab.com/de-de/Published-Articles/Teaching-DoE-with-Paper-Helicopters-and-Minitab/

***** Source: Minitab 17***


A
B
SigmaCopter
C
D
A
B
SigmaCopter
C
D


A
B
C
D
SigmaCopter

Screening of probable and Optimization of important Influences (X)



1.

DoE Screening Experiment: Identify the important Influences x for the Result Y

Design of Experiments (DoE)

Screening Design

Purpose

Evaluate 6-15 probably relevant Factors with 2 categorical (nominal) or continuous (cardinal) levels each (= Influences x) on one (or more) attributes of the Output (Y), to identify the important x for the succeeding Modelling Design.

Focus

Efficiency	An experiment based on a Screening Design is the most efficient method to identify the relevant x in a large number of x´s. Screening Designs are typically of resolution III or IV (fractional designs) which allow to identify significant main effects of many factors with an efficient number of runs without considering interaction effects.

Example

Identify important influences of Inputs, Methods and Resources (x) on attributes of (Y)

Identify important influences of your lifestyle (x) on your fitness (Y)

Identify important influences of the layout (x) of the sigmaCopter on its flight duration (Y)

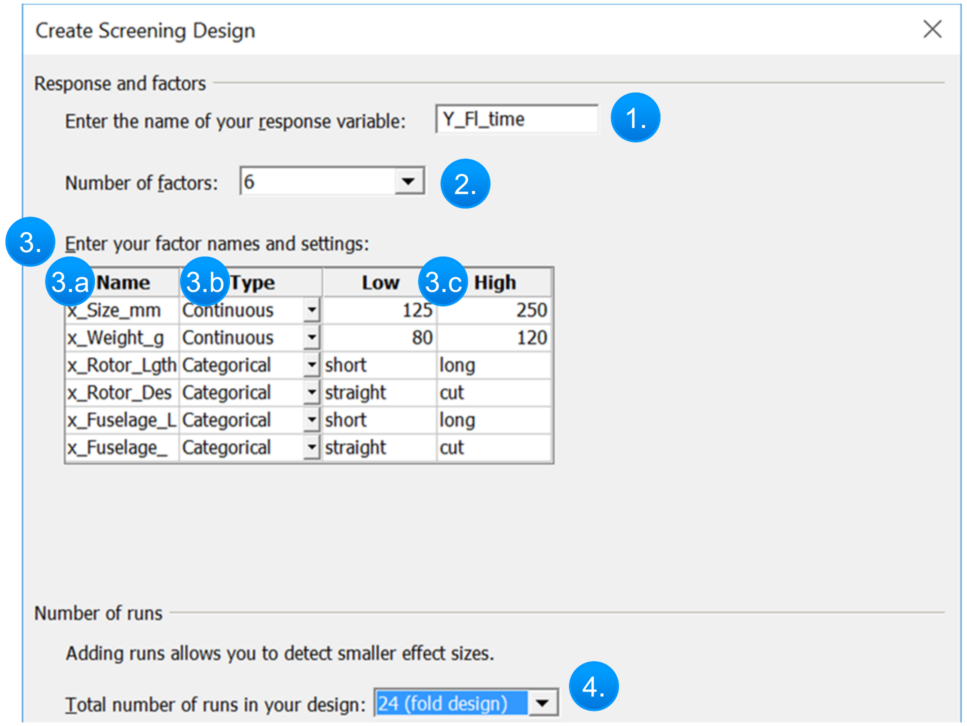
Y	Scale Level
1	cardinal
x	**Scale Level**
6-15	nominal or cardinal, slit in 2 levels each

Alternative in Minitab Stat Menu

Stat/ DoE/ Factorial

note …

Identify the probably relevant Influences (x) in a preceding Brainstorming / Graphical Analysis. Then specify the probably relevant range of the 2 Levels (low vs. high) of each selected Influence/ Factor (x).



Dialog: Create Screening Design (Worksheet)	
1.	Name of the Y-Variable (in the Worksheet) (arbitrary)
2.	Number of Factors/ Influences x (in the Worksheet)
3	Specification of Factors/ Influences x
3.a	Name of the x-Factors (arbitrary)
3.b	Type of the Factor: - Categorical (= nominal) (a categorical variable cannot be handled as continuous) - Continuous (=cardinal) (a continuous variable can be handled as categorical)
3.c	Range of the Factors/ Influences (x) that should be investigated: - Low: lower corner point for each Factor - High: upper corner point for each Factor
4.	Number of Runs (determines the sample size; the higher the number of Runs, the higher the Power, the smaller the Differences, that could be detected, if present)

Example: Evaluate six probably relevant design features (x) for the flight duration (Y)

1.

DoE Screening Experiment: Identify the important Influences x for the Result Y

	C1	C2	C3	C4	C5	C6	C7-T	C8-T	C9-T	C10-T	C11
	StdOrder	RunOrder	PtType	Blocks	x_Size_mm	x_Weight_g	x_Rotor_Lgth	x_Rotor_Des	x_Fuselage_L	x_Fuselage_D	Y_Fl_time
1	14	1	1	1	125	80	long	straight	long	cut	6,0
2	23	2	1	1	250	80	long	cut	long	straight	4,8
3	11	3	1	1	125	120	short	straight	short	cut	4,4
4	12	4	1	1	125	80	short	straight	short	straight	5,3
5	24	5	1	1	250	120	long	cut	long	cut	3,9
6	17	6	1	1	125	80	long	straight	short	cut	5,9
7	6	7	1	1	250	120	long	straight	long	cut	4,0
8	8	8	1	1	125	80	long	cut	long	straight	5,8
9	5	9	1	1	250	120	short	cut	long	straight	3,3
10	18	10	1	1	125	80	short	cut	short	straight	5,2
11	22	11	1	1	125	120	long	cut	short	straight	4,7
12	20	12	1	1	250	120	short	straight	short	cut	3,4
13	13	13	1	1	125	120	short	cut	long	cut	4,4
14	10	14	1	1	250	80	short	straight	long	cut	4,5
15	9	15	1	1	125	80	short	cut	long	cut	5,4
16	4	16	1	1	250	80	long	cut	short	cut	4,8
17	1	17	1	1	250	80	long	straight	short	straight	4,8
18	7	18	1	1	125	120	long	cut	short	cut	4,8
19	19	19	1	1	250	80	short	straight	long	straight	4,4
20	16	20	1	1	125	120	short	straight	long	straight	4,4
21	21	21	1	1	250	120	long	straight	short	straight	3,8
22	15	22	1	1	250	80	short	cut	short	cut	4,3

Result: Created Worksheet	
C1 (StdOrder)	Standard Order resulting from systematic combination of all Factors and their Levels (experimental run).
C2 (RunOrder)	Randomized Standard Order to avoid sequence effects.
C3 (CenterPt or PtType)	Column with the point type. If you create a 2-level design, Minitab names this column CenterPt. If you create a Plackett-Burman or general full factorial design, Minitab names this column PtType. The codes are: 0 is a center point run and 1 is a corner point.
C4 (Blocks)	Column with the blocking variable. When the design is not blocked, Minitab sets all column values to 1.
C5 - Cn	Columns with the Factor-Level-Combinations for the experimental runs.
C11	empty Column for the measured Results of each experimental run (Factor-Level-Combination)

1.

Create Screening Design Report Card

Check	Status	Description
Randomization (1.a)	i	When you create a screening design, Minitab automatically randomizes the order of the experimental runs. Randomization balances the effect of uncontrollable conditions, such as changes to materials or personnel, and reduces the chance that these conditions will bias the results. When you conduct the screening experiment, make sure you perform the runs in random order as specified in the worksheet.
Next Steps (1.b)	i	To complete the screening process: 1. Complete all pre-experiment activities. For more information, view the Pre-Experiment Checklist. 2. Run your experiment in the order specified in the worksheet and collect the response data. 3. Enter the response data in column C11. 4. Fit the screening model. 5. Identify the critical few factors (5 or fewer) to include in the modeling design.

2.

Create Screening Design Summary Report

Experimental Goal

Reduce the number of factors down to the critical few that have the greatest influence on the response.

2.a

Design Information

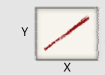
Response	Y_Fl_time
Base design	6 factors, 12 runs
Total runs	24

Factors and Settings

Factor	Low	High
x_Size_mm	125	250
x_Weight_g	80	120
x_Rotor_Lgth	short	long
x_Rotor_Des	straight	cut
x_Fuselage_L	short	long
x_Fuselage_D	straight	cut

Effect Estimation

This design will estimate the linear main effects for all factors. Interactions will not be estimated with this design.

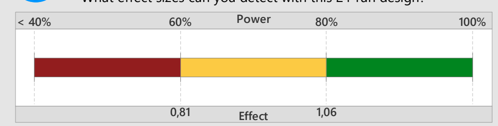


Main effect: Describes how the response (Y) changes if you change the setting of one factor (X).

2.b

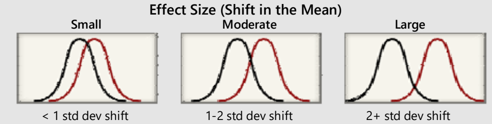
Detection Ability

What effect sizes can you detect with this 24-run design?



You have a 60% chance of detecting effects of 0,81 standard deviations or more and an 80% chance of detecting effects of 1,06.

Effect Size (Shift in the Mean)



Result: Report Card and Summary Report	
1. Report Card	Information about: a) Randomization StdOrder into RunOrder and b) next steps
2. Summary Report	Information about: a) the specified Design and b) Power of the experiment, i.e. probabilities to detect differences of a certain size, i.e. portions of standard deviations

Example: Evaluate six probably relevant design features (x) for the flight duration (Y)

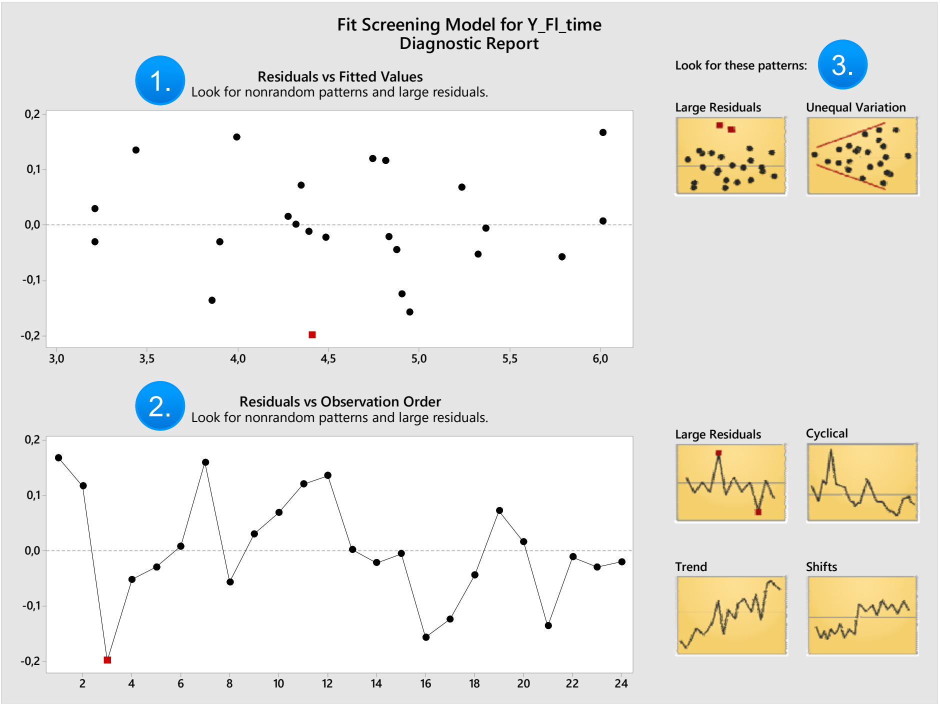
1.

DoE Screening Experiment: Identify the important Influences x for the Result Y

Fit Screening Model for Y_Fl_time
Report Card

Check	Status	Description
1. Unusual Data		One data point has a large residual and is not well fit by the model. This point is marked in red on the Diagnostic Report and is in row 3 of the worksheet. Because unusual data can have a strong influence on the results, try to identify the cause for its unusual nature. Correct any data entry or measurement errors. Consider performing trials associated with special causes again and redoing the analysis.
2. Randomization		When you create a designed experiment, Minitab automatically randomizes the order of the experimental runs. Randomization balances the effect of uncontrollable conditions, such as changes to materials or personnel, and reduces the chance that these conditions will bias the results. If you did not perform the runs in random order, consider repeating the experiment.
3. Next Steps		Your screening experiment identified 5 critical factors, which are represented by the blue bars in the Pareto chart. You can now use those factors in a modeling design to create a predictive model for the response. When you set the factor levels in the modeling design, it is common practice to set them closer together than in the screening design. This can increase the chances of identifying optimal settings for the critical factors.

Create modeling design.



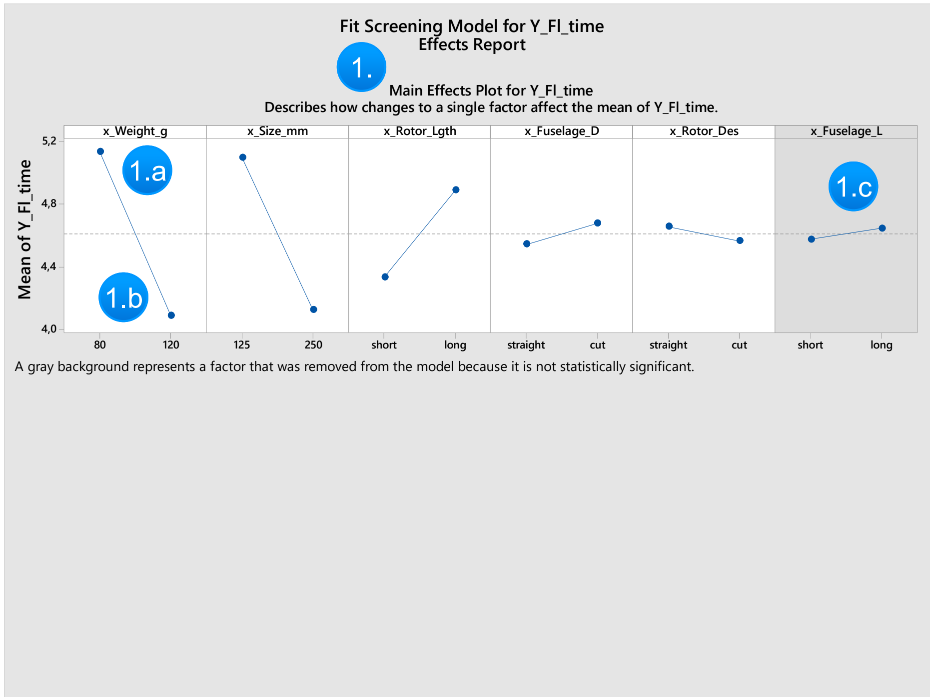
Result: Report Card	
1. Unusual Data	Warning of unusual data, to be seen in the Residuals Plots (next chart, right side)
2. Randomization	Information about advantage od Randomization of the Standard Order into the Run order
3. Next Steps	Summary of the Results of the Screening Experiment and advice for the selection of Factors and the range/ difference of their Levels.

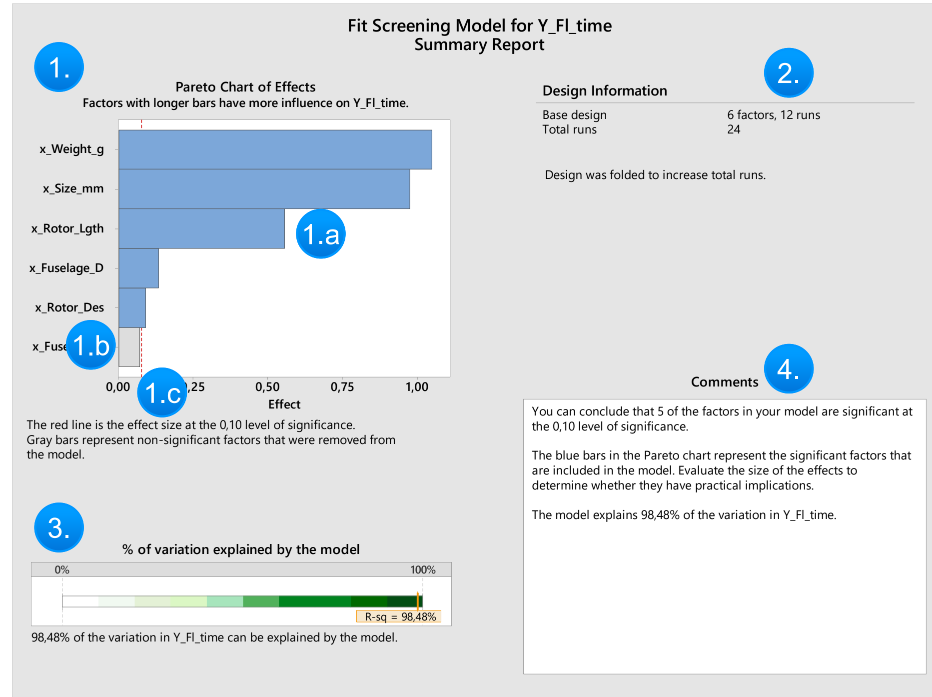
Result: Diagnostic Report	
1. Residuals vs. Fitted	Plot shows the Residuals, i.e. the deviation of the data points from the Factor Means, along the scale of Y, i.e. from small to large values of Y
1. Residuals vs. Observation Order	Plot shows the Residuals, i.e. the deviation of the data points from the Factor Means, along the Run Order of the collected data, as given in the Worksheet, i.e. from first to last collected data of Y
3. Signals	Signals as different patterns which show, that the Residuals are systematically influenced; try to identfy the Root-Causes of these patterns and eliminate them;

Example: Evaluate six probably relevant design features (x) for the flight duration (Y)

1.

DoE Screening Experiment: Identify the important Influences x for the Result Y





Result: Effects Report

1.	Main-Effects Plots for Y, showing the separated impact of each Factor (x) on Y, separated according to their specific Factor Levels; - white background means: Main-Effect of Factor is significant - grey background means: Main-Effect of Factor is not significant
1.a	Result of Factor: x_Weight, Level: 80g on: Y_Flight-time (while the impact of all other Factors is balanced)
1.b	Result of Factor: x_Weight, Level: 120g on: Y_Flight-time (while the impact of all other Factors is balanced)

Result: Summary Report

1. Pareto Chart of Effects	Factors (x) ranked according to their influence on Y, i.e. the Effect-Size, i.e. their influence on the Flight time
1.a	Blue bars indicate Factors (x) with significant influence (alpha= 10%)
1.b	Grey bars indicate Factors (x) with non-significant influence (alpha= 10%)
1.c	Red Line: shows the size of the Effect at the threshold of non-significant vs. significant results (alpha= 10%): all Effect Sizes right to this alpha-threshold are significant, all Effect Sizes left to this limit are not significant. If it is necessary, to also significantly identify Effect Sizes below this threshold, then a larger Sample Size is needed.
2. Design Information	Information of the analysed Design
3. Determination	Statement about R-square (R2), i.e. the % of variation in Y that can be explained by the model (= Factors and their Levels).
4. Comment	Summary and comments about results

Example: Evaluate six probably relevant design features (x) for the flight duration (Y)

2.

DoE Modeling Experiment: Identify Main-Effects, Interactions of x and optimize the Result Y

Design of Experiments (DoE)	
Modeling Design	
Purpose	
Evaluate 2-5 significant Factors (x) on the Result (Y), identified in the previous Screening Design, to find those Factor-Level combinations to a) maximize Y, b) minimize Y or c) achieve a target value of Y.	
Focus	
Effectivity	An experiment based on a Modeling Design is the most effective method to identify the optimal settings of a small number of x´s. Modeling Designs are typically full-factorial designs with as many replicates, which are necessary to identify Effect Sizes of a certain degree.
Example	
Optimize attributes of (Y) based on identified influences of Inputs, Methods and Resources (x)	
Optimize your fitness (Y) based on the identified influences on your lifestyle (x)	
Identify the Levels of the design features (x) for the optimal flight duration (Y)	

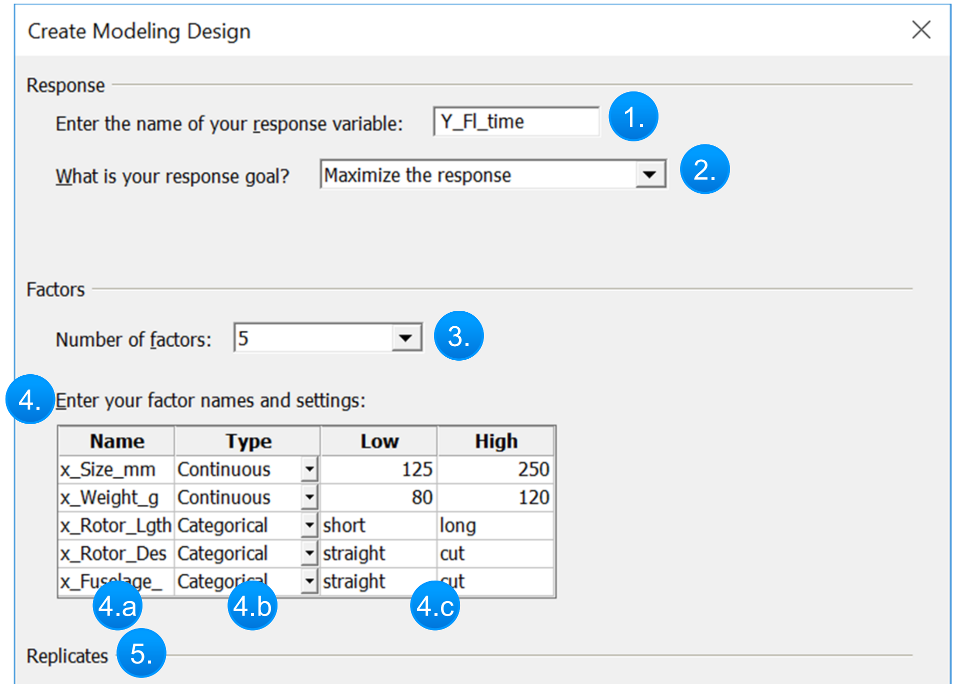
Y	Scale Level
1	cardinal
x	**Scale Level**
2-5	nominal or cardinal, split in 2 levels each

Alternative in Minitab Stat Menu

Stat/ DoE/ Factorial

note …

Verify the selected range of the 2 Levels (low vs. high) of each selected Influence/ Factor (x).



Dialog: Create Modeling Design (Worksheet)	
1.	Name of the Y-Variable (in the Worksheet)
2.	Goal for Y: a) maximize, b) minimize Response or c) achieve a target value for Y
3.	Number of Factors/ Influences x (in the Worksheet)
4.	Specification of Factors/ Influences x
4.a	Name of the x-Factors (arbitrary)
4.b	Type of the Factor: - Categorical (= nominal) (a categorical variable cannot be handled as continuous) - Continuous (=cardinal) (a continuous variable can be handled as categorical)
4.c	Range of the Factors/ Influences (x) that should be investigated: - Low: lower corner point for each Factor - High: upper corner point for each Factor
5.	Number of Replicates (multiple experimental runs with the same factor settings (levels) which increase the precision of the model)

Example: Identify the Levels of the design features (x) for the optimal flight duration (Y)

2.

DoE Modeling Experiment: Identify Main-Effects, Interactions of x and optimize the Result Y

↓	C1	C2	C3	C4	C5	C6	C7-T	C8-T	C9-T	C10
	StdOrder	RunOrder	CenterPt	Blocks	x_Size_mm	x_Weight_g	x_Rotor_Lgth	x_Rotor_Des	x_Fuselage_D	Y_Fl_time
1	7	1	1	1	125,0	120	long	straight	cut	4,85
2	16	2	1	1	250,0	120	long	cut	cut	3,74
3	1	3	1	1	125,0	80	short	straight	cut	5,21
4	13	4	1	1	125,0	80	long	cut	cut	5,58
5	12	5	1	1	250,0	120	short	cut	straight	2,94
6	30	6	0	1	187,5	100	long	straight	cut	3,95
7	22	7	0	1	187,5	100	long	straight	cut	3,96
8	26	8	0	1	187,5	100	long	straight	straight	3,69
9	10	9	1	1	250,0	80	short	cut	cut	4,28
10	19	10	0	1	187,5	100	short	cut	straight	3,26
11	2	11	1	1	250,0	80	short	straight	straight	4,03
12	21	12	0	1	187,5	100	short	straight	cut	3,21
13	9	13	1	1	125,0	80	short	cut	straight	5,17
14	20	14	0	1	187,5	100	long	cut	straight	3,71
15	11	15	1	1	125,0	120	short	cut	cut	4,38
16	8	16	1	1	250,0	120	long	straight	straight	3,66
17	24	17	0	1	187,5	100	long	cut	cut	3,69
18	17	18	0	1	187,5	100	short	straight	straight	3,13
19	28	19	0	1	187,5	100	long	cut	straight	3,65
20	23	20	0	1	187,5	100	short	cut	cut	3,20

Result: Created Worksheet	
C1 (StdOrder)	Standard Order resulting from systematic combination of all Factors and their Levels (experimental run).
C2 (RunOrder)	Randomized Standard Order to avoid sequence effects.
C3 (CenterPt or PtType)	Column with the point type. If you create a 2-level design, Minitab names this column CenterPt. If you create a Plackett-Burman or general full factorial design, Minitab names this column PtType. The codes are: 0 is a center point run and 1 is a corner point.
C4 (Blocks)	Column with the blocking variable. When the design is not blocked, Minitab sets all column values to 1.
C5 - Cn	Columns with the Factor-Level-Combinations for the experimental runs.
C10	empty Column for the measured Results of each experimental run (Factor-Level-Combination)

1.

Create Modeling Design
Report Card

Check	Status	Description
Randomization (1.a)	i	When you create a modeling design, Minitab automatically randomizes the order of the experimental runs. Randomization balances the effect of uncontrollable conditions, such as changes to materials or personnel, and reduces the chance that these conditions will bias the results. When you conduct the modeling experiment, make sure you perform the runs in random order as specified in the worksheet.
Next Steps (1.b)	i	To complete the optimization process: 1. Complete all pre-experiment activities. For more information, view the Pre-Experiment Checklist. 2. Run your experiment in the order specified in the worksheet and collect the response data. 3. Enter the response data in column C10. 4. Fit the linear model. 5. If curvature is significant, add points for curvature, collect the response data, and fit a quadratic model.

2.

Create Modeling Design
Summary Report

Experimental Goal

Construct a model that describes the relationship between the response and critical factors. If the model is adequate, use it to find optimal settings for the factors.

2.a

Design Information

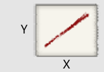
Response	Y_Fl_time
Goal	Maximize
Base design	5 factors, 16 runs
Replicates	1
Center points	16
Total runs	32

Factors and Settings

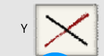
Factor	Low	High
x_Size_mm	125	250
x_Weight_g	80	120
x_Rotor_Lgth	short	long
x_Rotor_Des	straight	cut
x_Fuselage_D	straight	cut

Effect Estimation

This design will estimate all linear main effects and two-way interactions.



Main effect: Describes how the response (Y) changes if you change the setting of one factor (X).

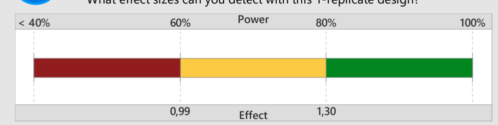


Interaction: Describes how the response (Y) changes if you change the settings of two factors (X).

2.b

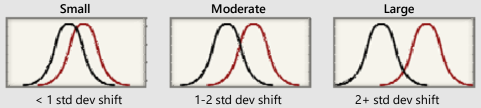
Detection Ability

What effect sizes can you detect with this 1-replicate design?



You have an 80% chance of detecting effects of 1,30 standard deviations or more. With 2 replicates, you can detect effects of 0,89.

Effect Size (Shift in the Mean)

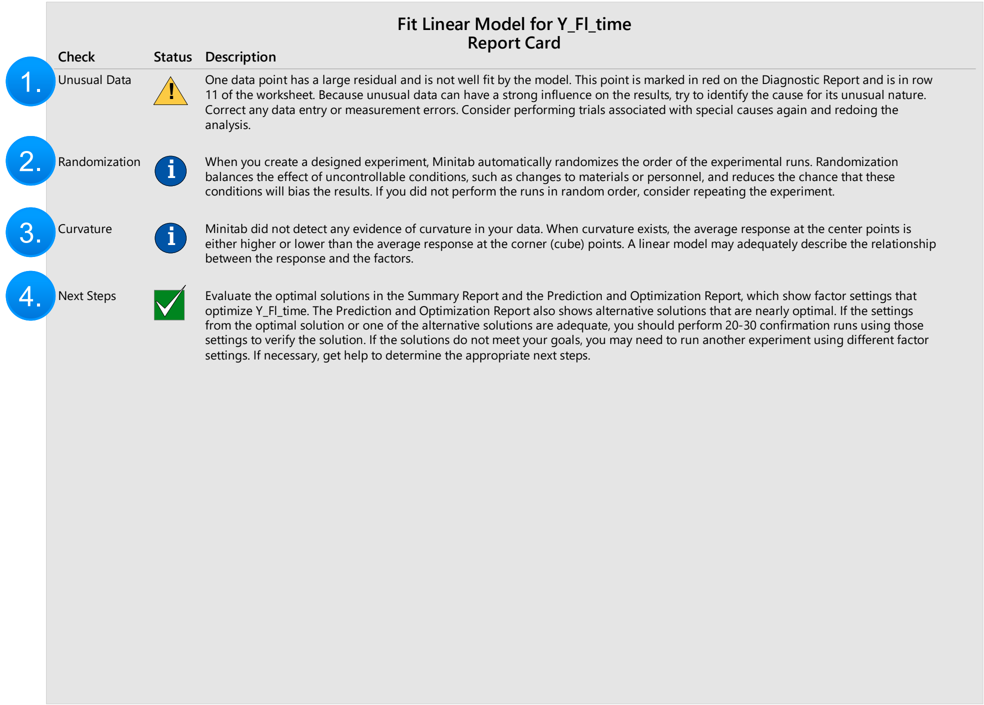


Result: Report Card and Summary Report	
1. Report Card	Information about: a) Randomization StdOrder into RunOrder and b) next steps
2. Summary Report	Information about: a) the specified Design and b) Power of the experiment, i.e. probabilities to detect differences of a certain size, i.e. portions of standard deviations

Example: Identify the Levels of the design features (x) for the optimal flight duration (Y)

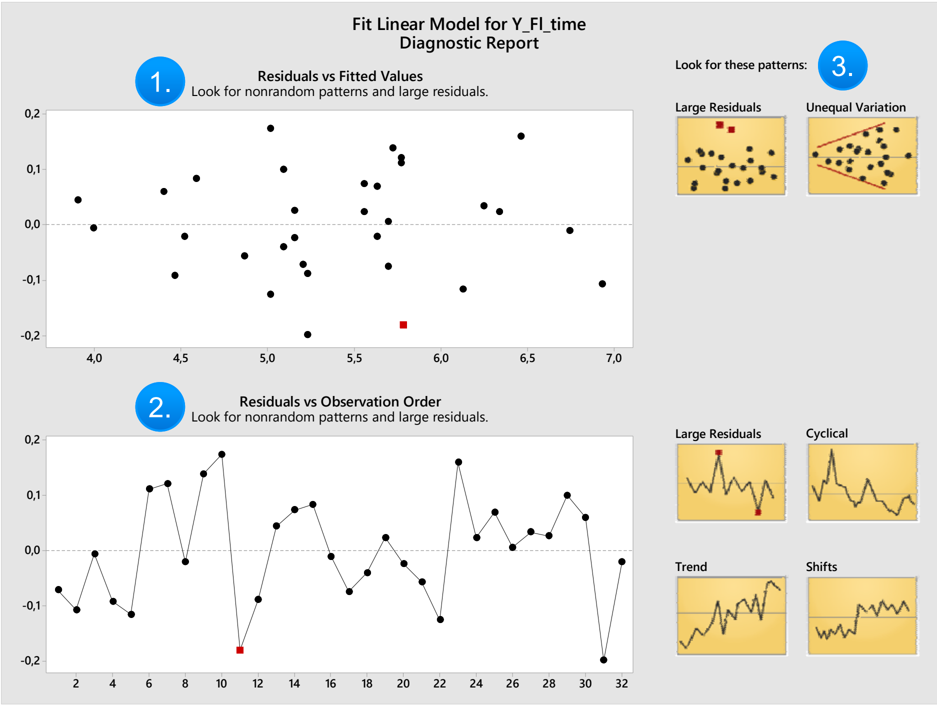
2.

DoE Modeling Experiment: Identify Main-Effects, Interactions of x and optimize the Result Y



Fit Linear Model for Y_Fl_time
Report Card

	Check	Status	Description
1.	Unusual Data		One data point has a large residual and is not well fit by the model. This point is marked in red on the Diagnostic Report and is in row 11 of the worksheet. Because unusual data can have a strong influence on the results, try to identify the cause for its unusual nature. Correct any data entry or measurement errors. Consider performing trials associated with special causes again and redoing the analysis.
2.	Randomization		When you create a designed experiment, Minitab automatically randomizes the order of the experimental runs. Randomization balances the effect of uncontrollable conditions, such as changes to materials or personnel, and reduces the chance that these conditions will bias the results. If you did not perform the runs in random order, consider repeating the experiment.
3.	Curvature		Minitab did not detect any evidence of curvature in your data. When curvature exists, the average response at the center points is either higher or lower than the average response at the corner (cube) points. A linear model may adequately describe the relationship between the response and the factors.
4.	Next Steps		Evaluate the optimal solutions in the Summary Report and the Prediction and Optimization Report, which show factor settings that optimize Y_Fl_time. The Prediction and Optimization Report also shows alternative solutions that are nearly optimal. If the settings from the optimal solution or one of the alternative solutions are adequate, you should perform 20-30 confirmation runs using those settings to verify the solution. If the solutions do not meet your goals, you may need to run another experiment using different factor settings. If necessary, get help to determine the appropriate next steps.



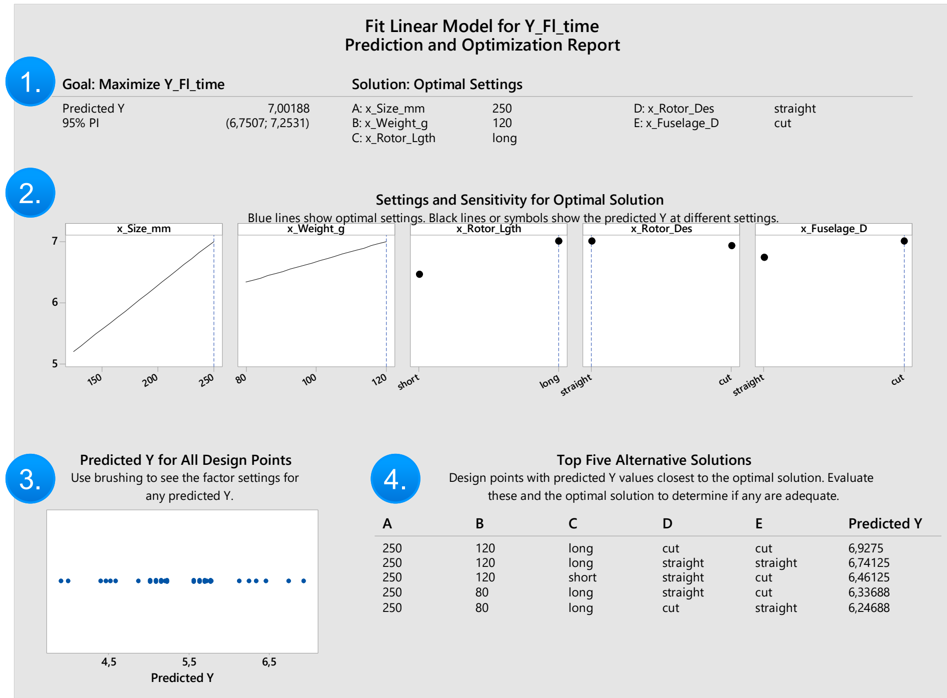
Result: Report Card	
1. Unusual Data	Warning of unusual data, to be seen in the Residuals Plots (next chart, right side)
2. Randomization	Information about advantage od Randomization of the Standard Order into the Run order
3. Curvature	Detection of Curvature means: non linear Influence of a Factor (x) on the Result (Y), based on the Results of the Center Point (x) in Relation to the Corner Points (x); Curvature would make a Response Surface Design necessary;
4. Next Steps	Summary of the Results of the Modeling Experiment and advices for validating the Result.

Result: Diagnostic Report	
1. Residuals vs. Fitted	Plot shows the Residuals, i.e. the deviation of the data points from the Factor Means, along the scale of Y, i.e. from small to large values of Y
1. Residuals vs. Observation Order	Plot shows the Residuals, i.e. the deviation of the data points from the Factor Means, along the Run Order of the collected data, as given in the Worksheet, i.e. from first to last collected data of Y
3. Signals	Signals as different patterns which show, that the Residuals are systematically influenced; try to identfy the Root-Causes of these patterns and eliminate them;

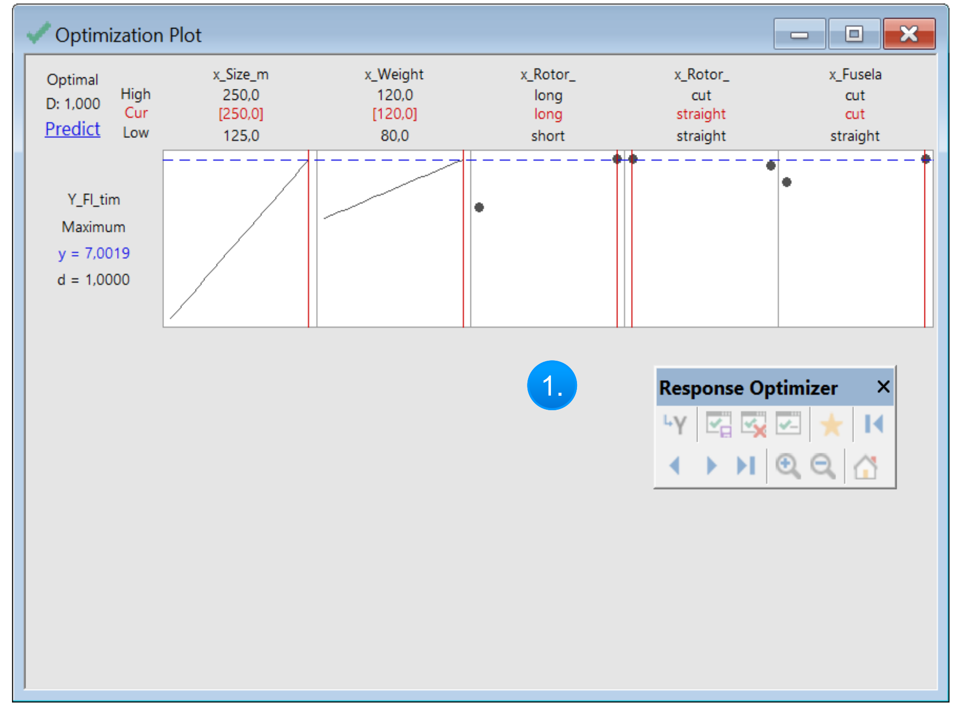
Example: Identify the Levels of the design features (x) for the optimal flight duration (Y)

2.

DoE Modeling Experiment: Identify Main-Effects, Interactions of x and optimize the Result Y



A	B	C	D	E	Predicted Y
250	120	long	cut	cut	6,9275
250	120	long	straight	straight	6,74125
250	120	short	straight	cut	6,46125
250	80	long	straight	cut	6,33688
250	80	long	cut	straight	6,24688



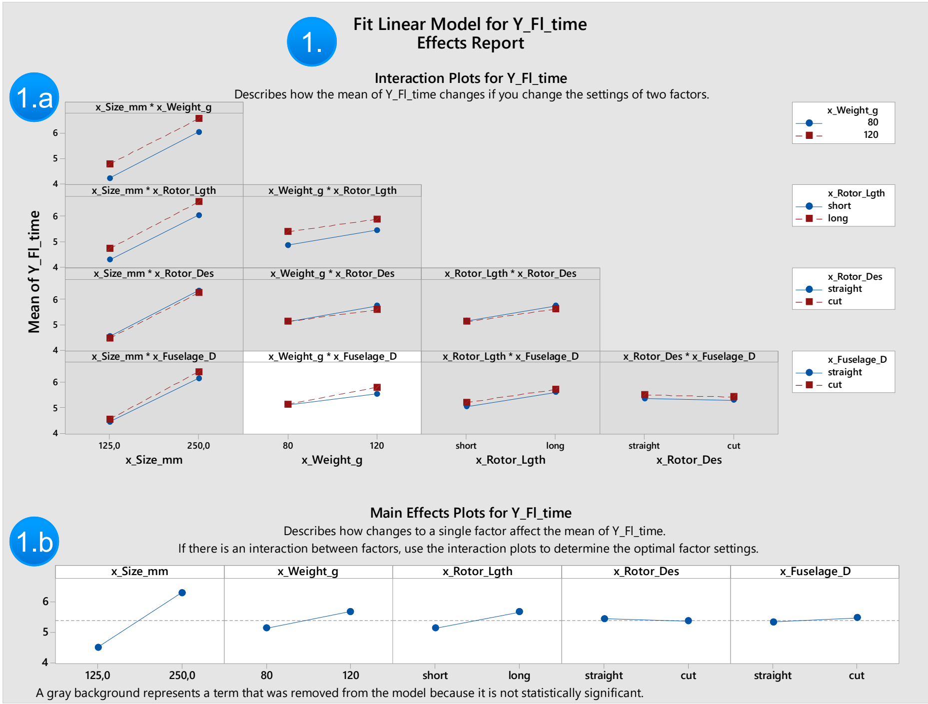
Result: Prediction and Optimization Report	
1. Optimal Settings	Optimal settings of all significant Factors, i.e. their Levels for the selected goal of Y (a. maximize, minimize or achieve a target value)
2. Settings and Sensitivity	Graphical Display of optimal settings, with the blue lines indicating the selected Factor-Level (x) for the optimal result (Y)
3. Predicted Y	Predicted values for Y for all Design Points, i.e. all Factor-Level combinations (x)
4. Alternative Solutions	Ranking of alternative solutions to the optimal solution, which might have advantages, not investigated in the experiments (e.g. costs)

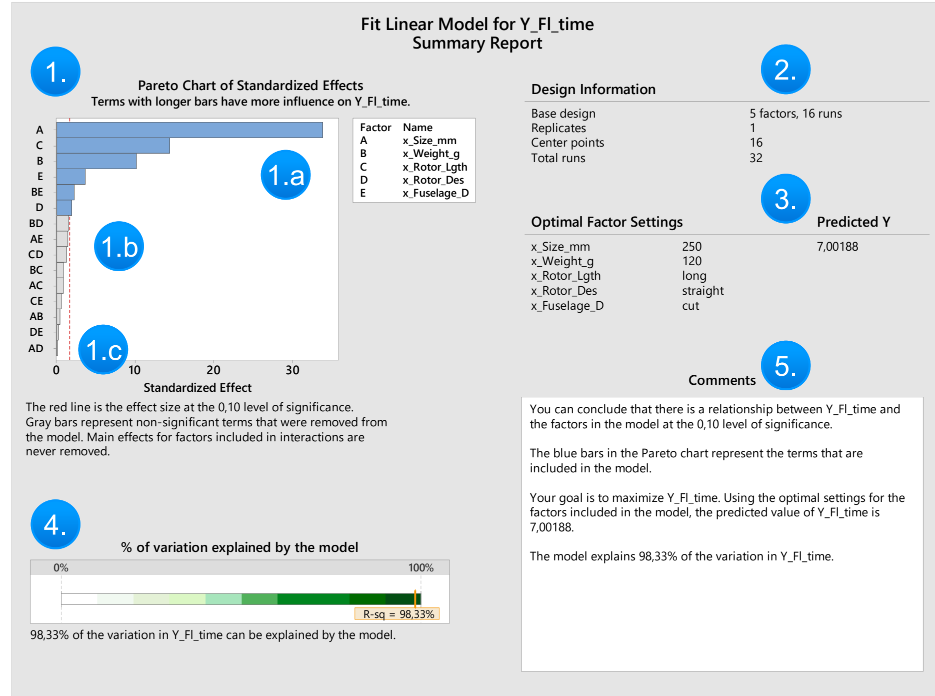
Alternative: Response Optimizer	
1. Menu: Stat/ DoE/ Factorial/ Response Optimizer	A more flexible tool to find the optimal solution and systematically identify alternative solutions, based on the given Results is the Response Optimizer. It is immediately available after the results in the Prediction and Optimization Report have been calculated.

Example: Identify the Levels of the design features (x) for the optimal flight duration (Y)

2.

DoE Modeling Experiment: Identify Main-Effects, Interactions of x and optimize the Result Y





Result: Effects Report	
1.	Main-Effects Plots and Interaction Plots for Y, showing the separated and combined impacts of Factors (x) on Y; - white background means: Effect is significant - grey background means: Effect is not significant
1.a	Interaction Plots for all Factor x Factor combinations and their Levels
1.b	Main-Effects Plots for each Factor x and their Levels

Result: Summary Report	
1. Pareto Chart of Standardized Effects	Factors (x) and Interaction of Factors ranked according to their influence on Y, i.e. the Standardized Effect-Size, i.e. their influence on the Flight time
1.a	Blue bars indicate Factors and Interactions with significant influence (alpha= 10%)
1.b	Grey bars indicate Factors and Interactions with non-significant influence
1.c	Red Line: shows the size of the Effect at the threshold of non-significant vs. significant results (alpha= 10%): all Effect Sizes right to this alpha-threshold are significant, all Effect Sizes left to this limit are not significant. If it is necessary, to also significantly identify Effect Sizes below this threshold, then a larger Sample Size is needed.
2. Design Information	Information of the analyzed Design
3. Optimal Settings	Optimal settings for the Factors (x) and the corresponding predicted Result for Y
4. Determination	R-square (R2), i.e. the % of variation in Y that can be explained by the Factors (x)
5. Comment	Summary and comments about results

Example: Identify the Levels of the design features (x) for the optimal flight duration (Y)

Literature

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end of course

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